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Supertransient Magnetohydrodynamic Turbulence in Accretion Disks

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OUTLINE

- Motivation: transition to turbulence in keplerian disks
- The shearing box model
- Subcritical transition to turbulence and supertransients in Keplerian disks
- Conclusions

ORIGINAL MOTIVATION

A&A 476, 1123–1132 (2007)
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Astronomy & Astrophysics

MHD simulations of the magnetorotational instability in a shearing box with zero net flux

II. The effect of transport coefficients

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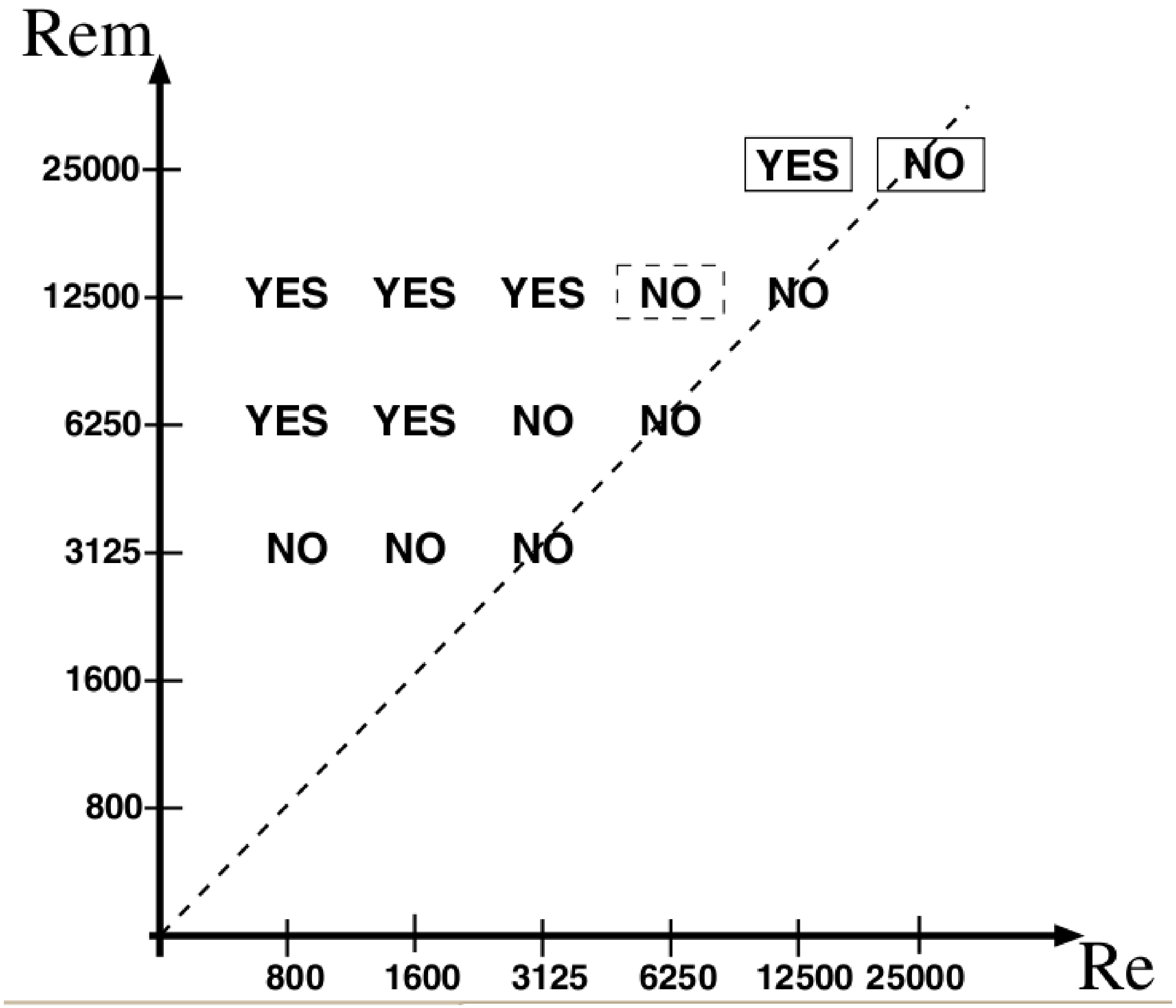
TRANSITION TO TURBULENCE



Source: Fromang, Papaloizou, Lesur, and Heinemann, A&A 476, 1123-1132 (2007).

KEPLERIAN DISKS

Kepler's third law of planetary motion: The period T of a planetary orbit is proportional to the three-halves power of its mean distance r from the Sun.

$T \mu r^{3/2}$

$1/T \mu r^{-3/2}$

$\Omega(r) \mu r^{-3/2}$

Thus, angular velocity increases with decreasing radius.

$\mathbf{L} = \mathbf{r} \times m\mathbf{v}$

$= \mathbf{r} \times m(\mathbf{\Omega} \times \mathbf{r})$, $\quad \mathbf{\Omega} = \Omega\hat{\mathbf{z}}$

$|L| \mu r^2\Omega$

$\mu r^2 r^{-3/2}$

Thus, angular momentum decreases with decreasing radius.
For matter to move inward, its angular momentum must decrease. Conservation of net angular momentum requires that the excess is transferred outward.

RATE OF ACCRETION DEPENDS ON TRANSPORT OF ANGULAR MOMENTUM

KEPLERIAN DISKS

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The new solution: a dynamo mechanism to prevent the quenching of **B** and sustain the turbulence.

MODELING KEPLERIAN SHEAR FLOWS

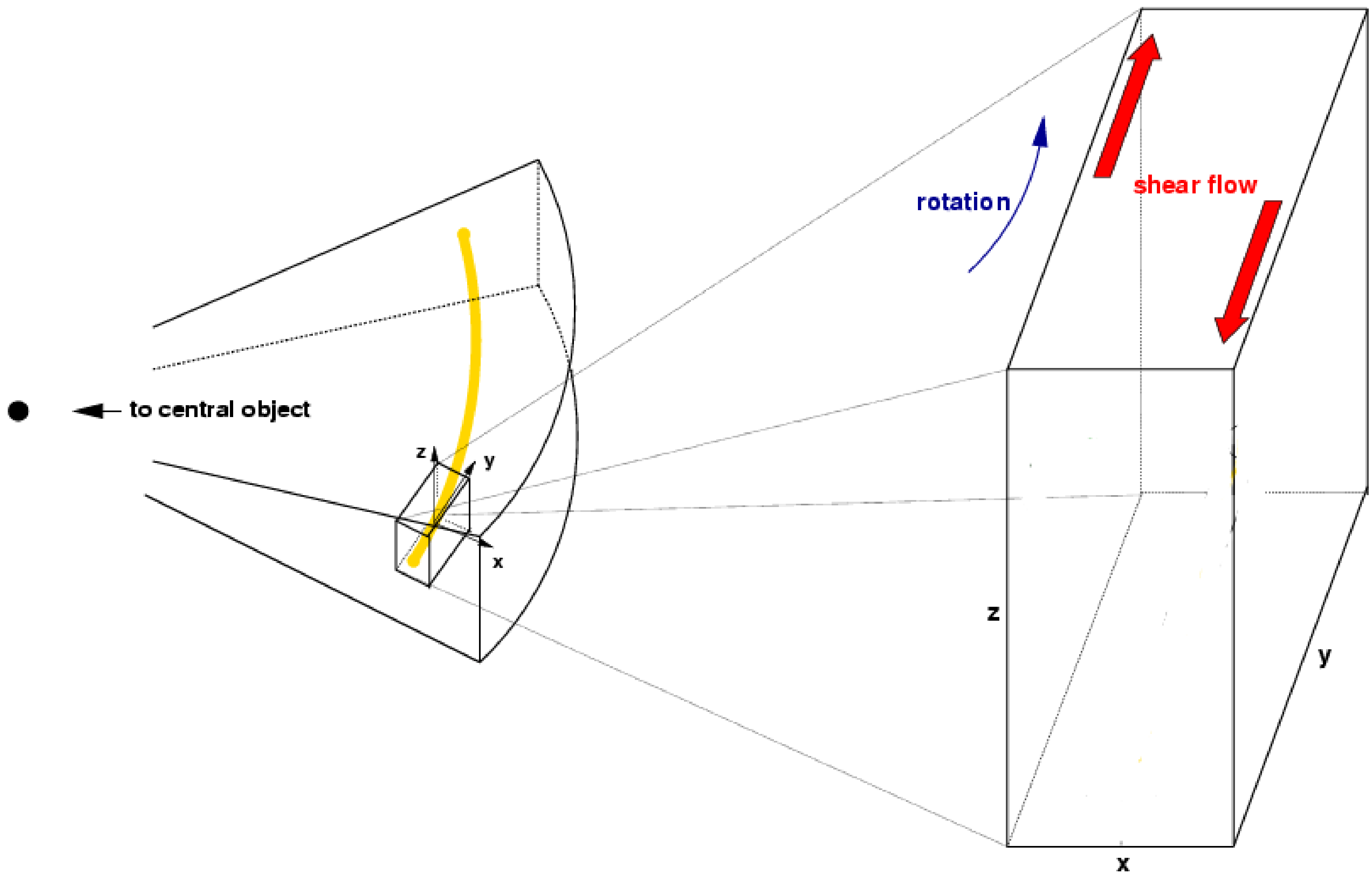


Image credit: Ziegler, A&A 367, 170-182 (2001).

MODELING KEPLERIAN SHEAR FLOWS

In a box moving with angular velocity $\Omega_0 = \Omega(r_0)$, with $\phi \rightarrow y$ and $r \rightarrow r_0 + x$, the incompressible MHD equations read as:

$$\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho} \nabla P + \frac{1}{\mu_0 \rho} (\nabla \times \mathbf{B}) \times \mathbf{B} - 2\boldsymbol{\Omega}_0 \times \mathbf{v} + 2\Omega_0 S x \hat{\mathbf{x}} + \nu \nabla^2 \mathbf{v},$$

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B},$$

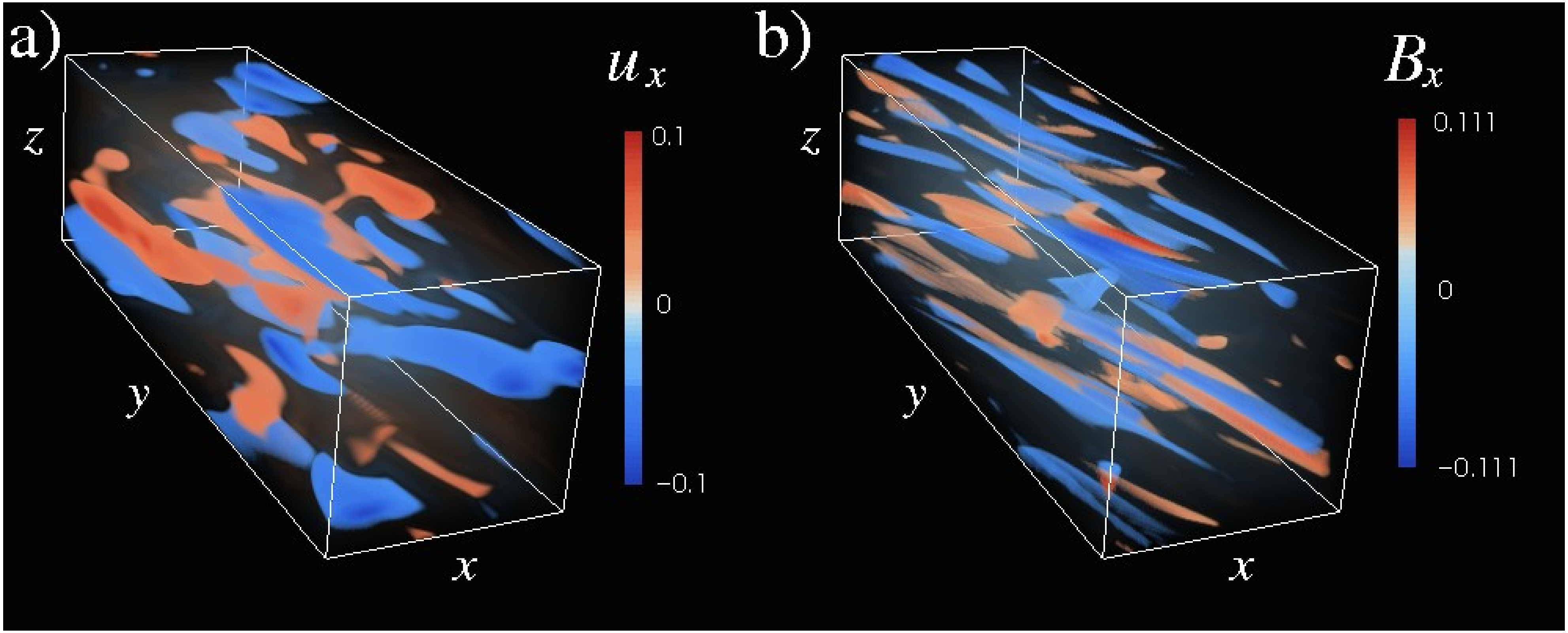
$$\nabla \cdot \mathbf{B} = 0, \qquad \nabla \cdot \mathbf{v} = 0,$$

xyz: $1 \times \pi \times 1$
μ_0 = magnetic permeability
Re = Sd^2/ν
Rm = Sd^2/η
$S = -r\partial_r \Omega_0 = (3/2)\Omega_0$
d = shearwise size of the box

We set:
$\Omega_0 = 2/3$
$\nu = 3.2\text{e-}4 \rightarrow \text{Re} = 3125$
Rm is the control parameter
Shearing sheet boundary conditions

The Snoopy code: pseudospectral method (http://ipag.osug.fr/~glesur/snoopy.html)

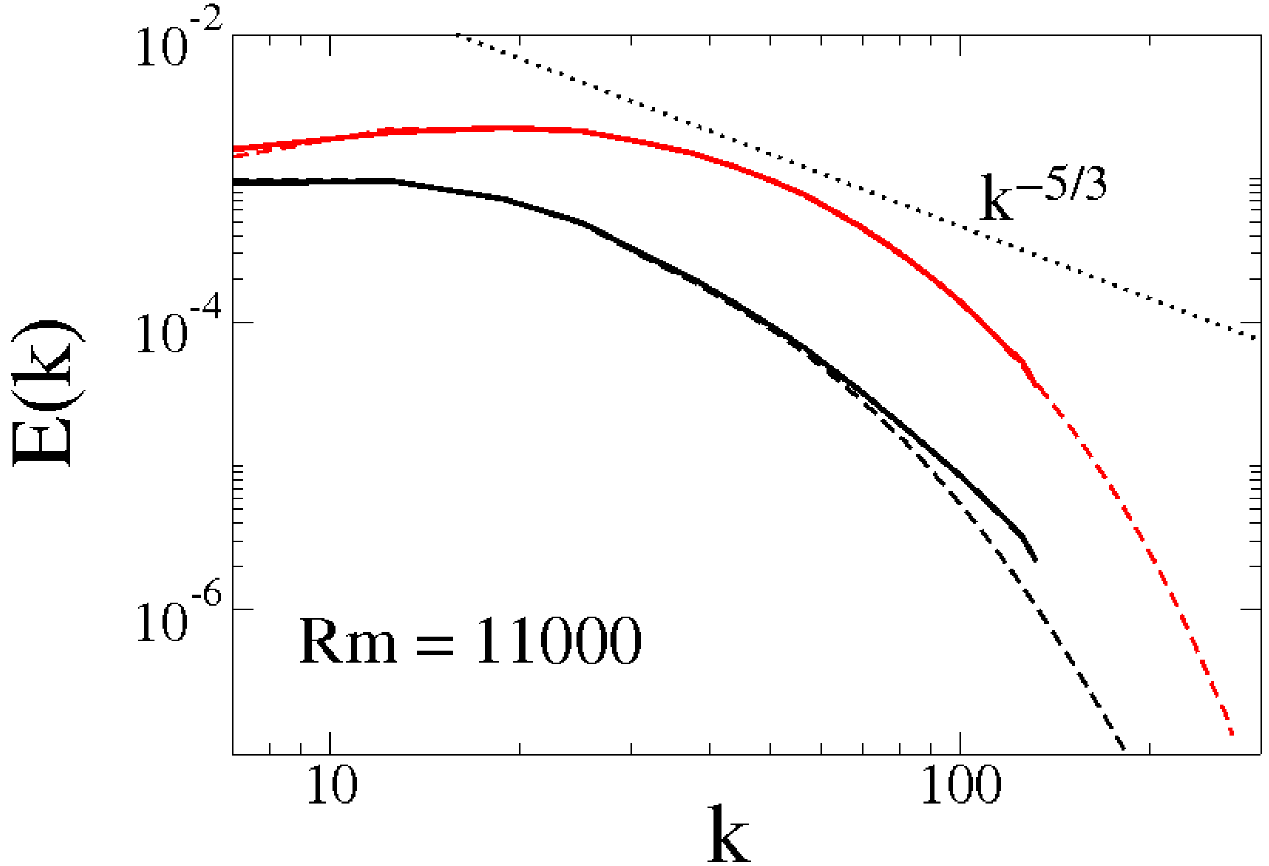
KEPLERIAN SHEAR FLOWS

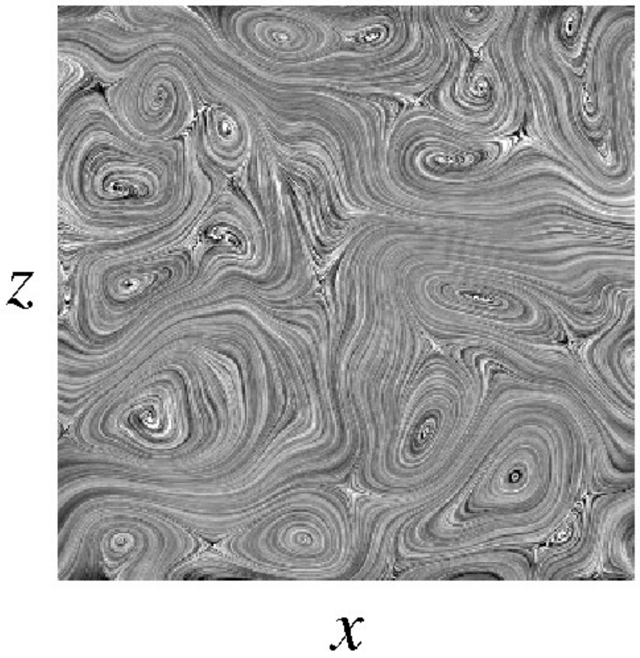


- Volume rendering plots of u_x and B_x for Re=3125 and Rm=11000.
- $\mathbf{u} = \mathbf{v} - \mathbf{v}_0$, $\mathbf{v}_0 = -Sx\hat{\mathbf{y}}$
- **Obs.:** For Re = LU/ν and Rm = LU/η, Re < 180 and Rm < 650.

Source: Rempel, Lesur and Proctor, Phys. Rev. Lett. 105,044501 (2010).

KEPLERIAN SHEAR FLOWS





- Kinetic (Black) and magnetic (red) power spectra for 64x128x64 (solid line) and 128x256x126 (dashed line) for Re=3125 and Rm=11000.

Source: Rempel, Lesur and Proctor, Phys. Rev. Lett. 105,044501 (2010).

KEPLERIAN SHEAR FLOWS

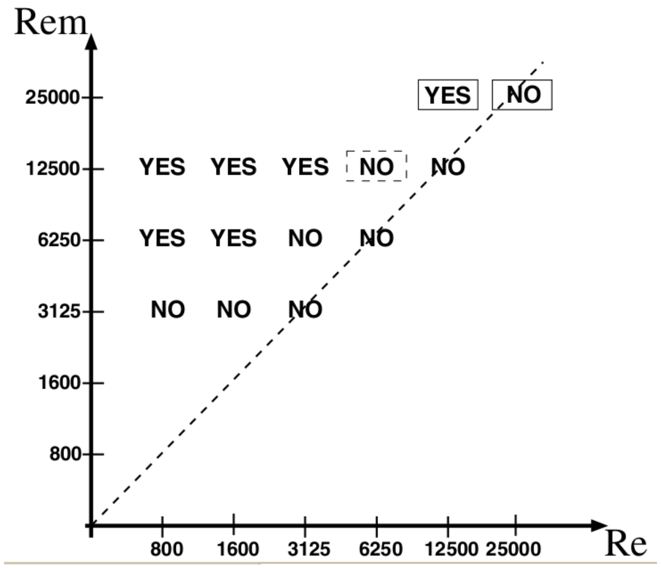
TABLE I. Time-averaged quantities for $R_m = 12\,500$.

Res.	$64 \times 128 \times 64$	$128 \times 256 \times 128$
$\langle E_{\mathrm{kin}} \rangle_t$	2.11×10^{-3}	2.07×10^{-3}
$\langle E_{\mathrm{mag}} \rangle_t$	8.45×10^{-3}	8.52×10^{-3}
$\langle \alpha_{\mathrm{Rey}} \rangle_t$	5.22×10^{-4}	5.24×10^{-4}
$\langle \alpha_{\mathrm{Max}} \rangle_t$	3.68×10^{-3}	3.69×10^{-3}

- $E_{kin} = 0.5<u^2>^2$
- $E_{mag} = 0.5<B^2>^2$
- $\alpha_{rey} = <u_x u_y>$
- $\alpha_{max} = -<B_x B_y>$

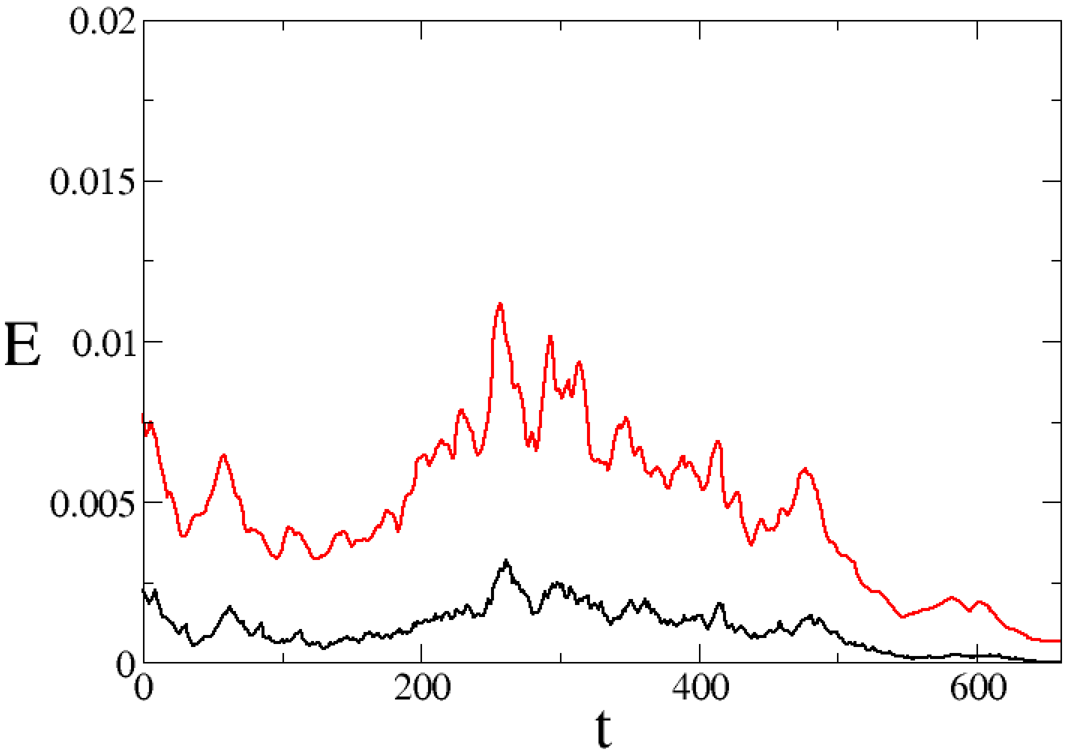
Source: Rempel, Lesur and Proctor, Phys. Rev. Lett. 105,044501 (2010).

KEPLERIAN SHEAR FLOWS

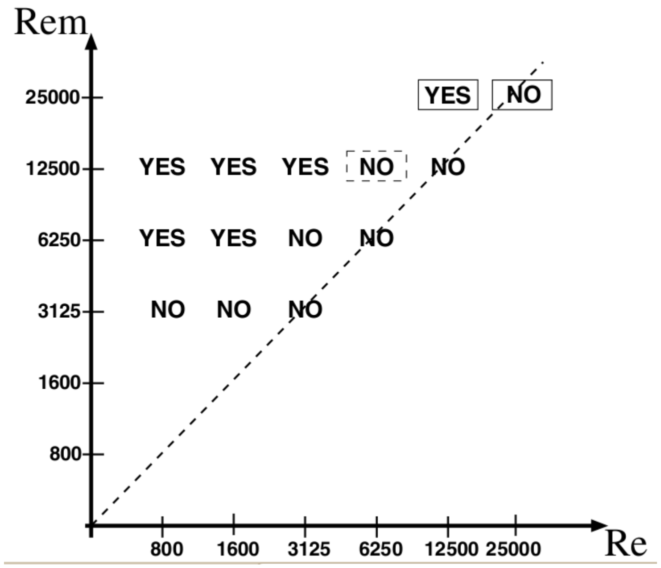


Re = 3125

Rem = 10000



KEPLERIAN SHEAR FLOWS



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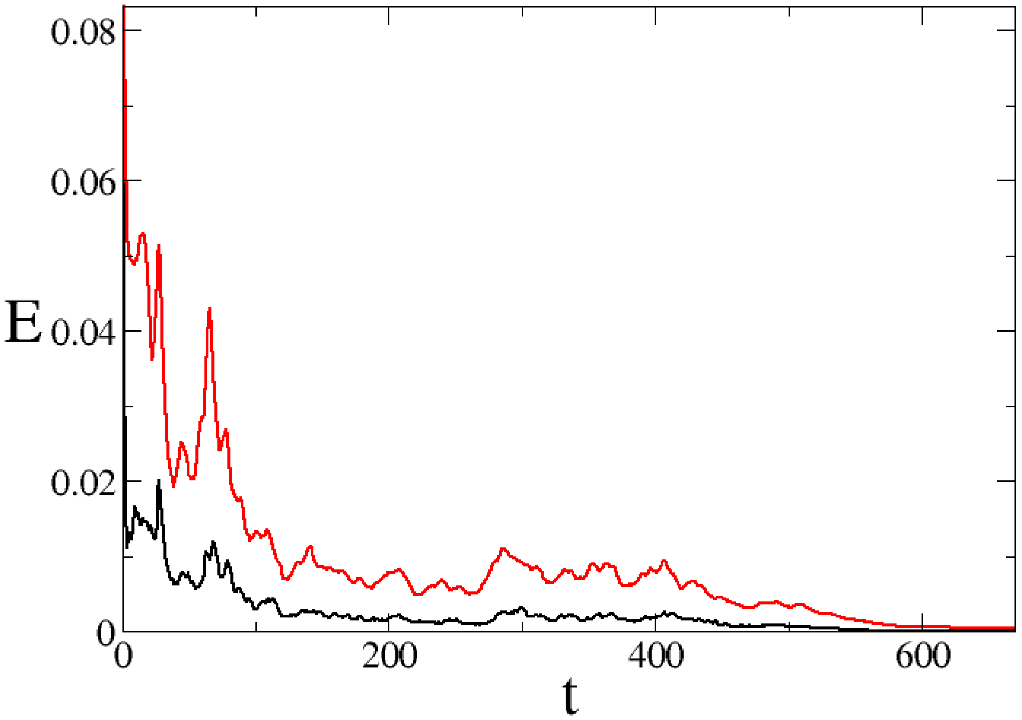
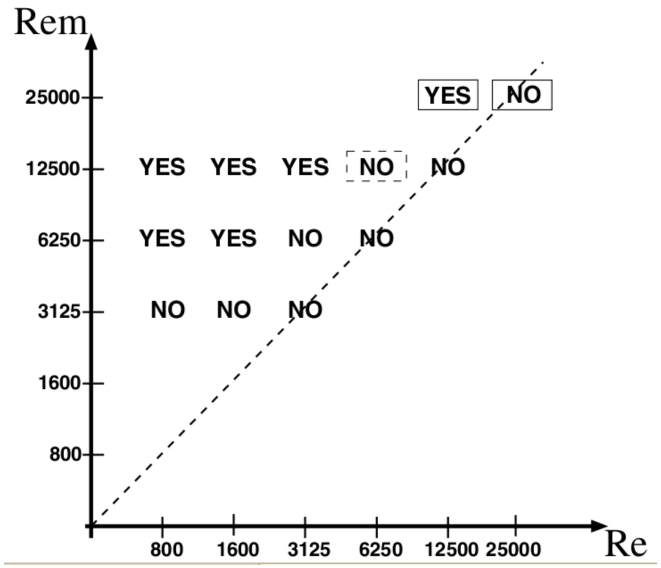
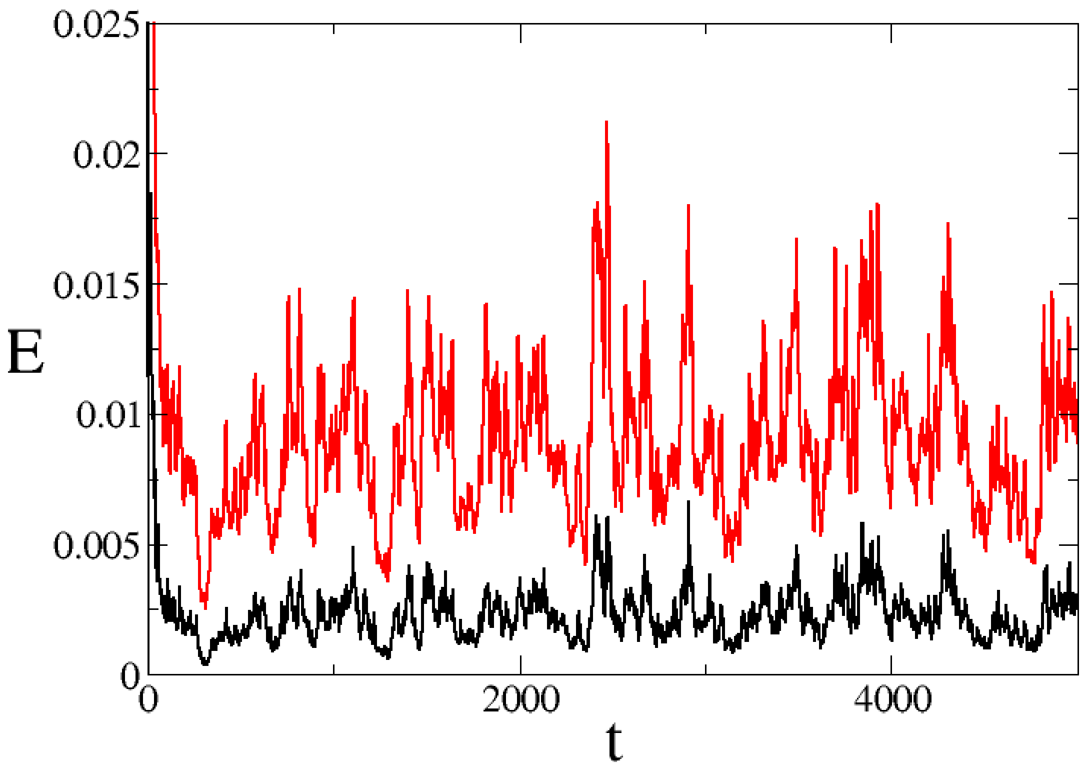


KEPLERIAN SHEAR FLOWS

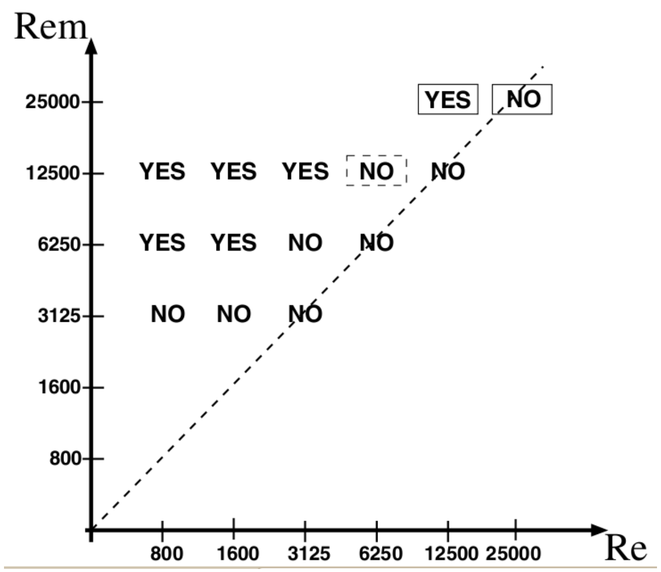


Re = 3125

Rem = 12500



KEPLERIAN SHEAR FLOWS



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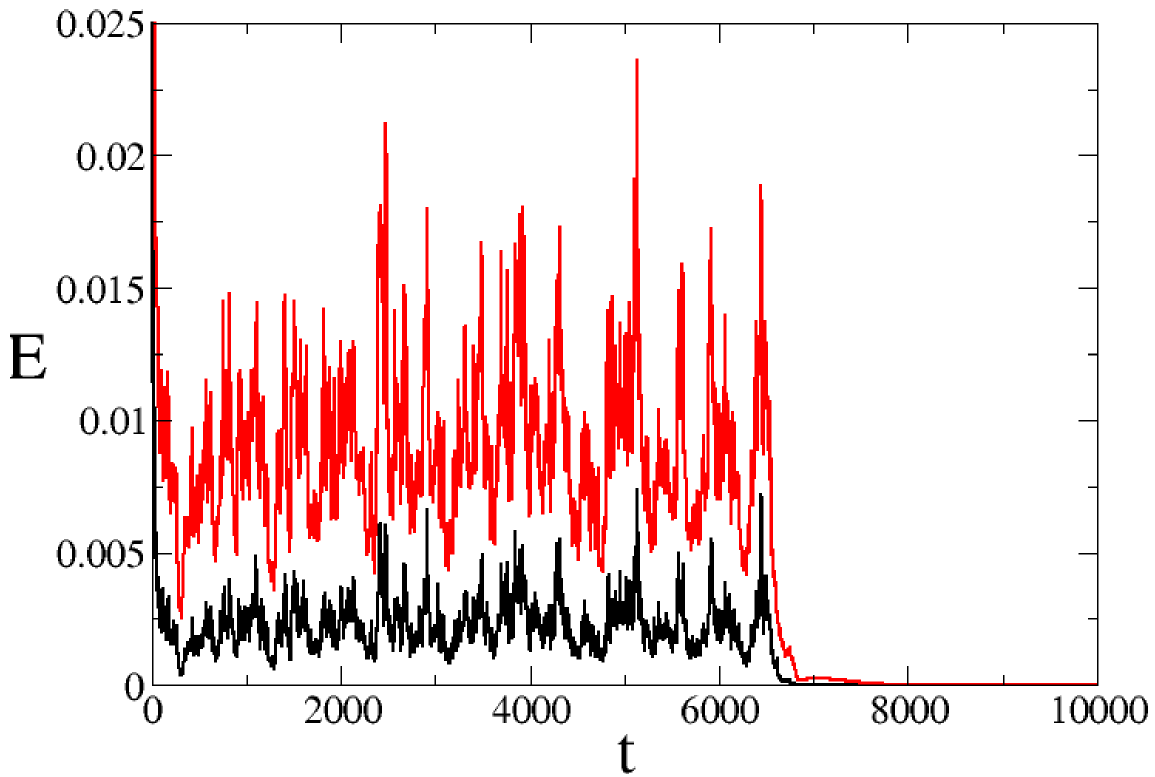


SUPERTRANSIENTS IN SPATIOTEMPORAL SYSTEMS

Transient times increase rapidly with control parameter (e. g. system size L)

- **Type-I supertransient (power-law scaling):**

  $\tau(L) \sim L^{\alpha}, \qquad \alpha > 0$

  Lyapunov exponent and spatial deffects decay gradually with time

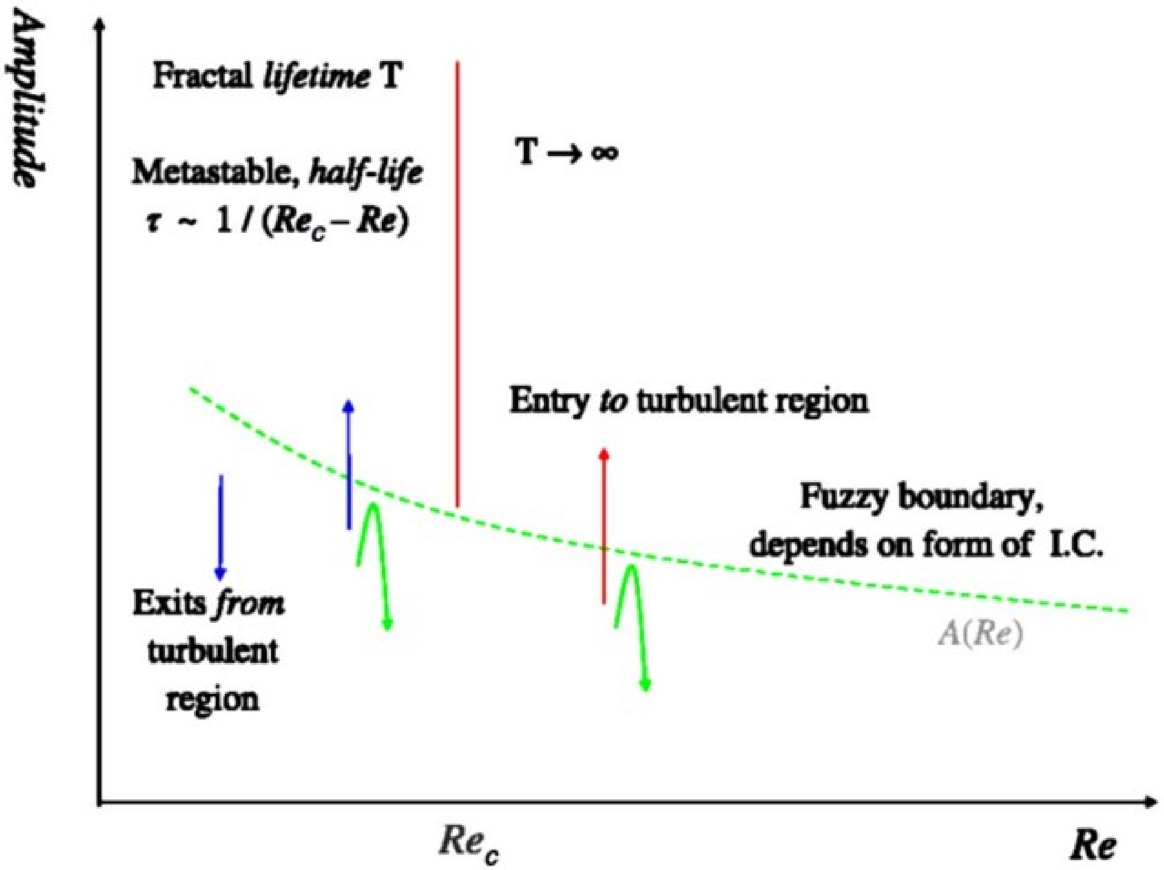
- **Type-II supertransient (exponential scaling):**

  $\tau(L) \sim \exp[cL^{\alpha}], \qquad c > 0, \alpha > 0$

  Statistically steady in time, transition to an attractor is abrupt. Due to chaotic saddles

Source: Tél and Lai, Phys. Rep. 460, 245-275 (2008)

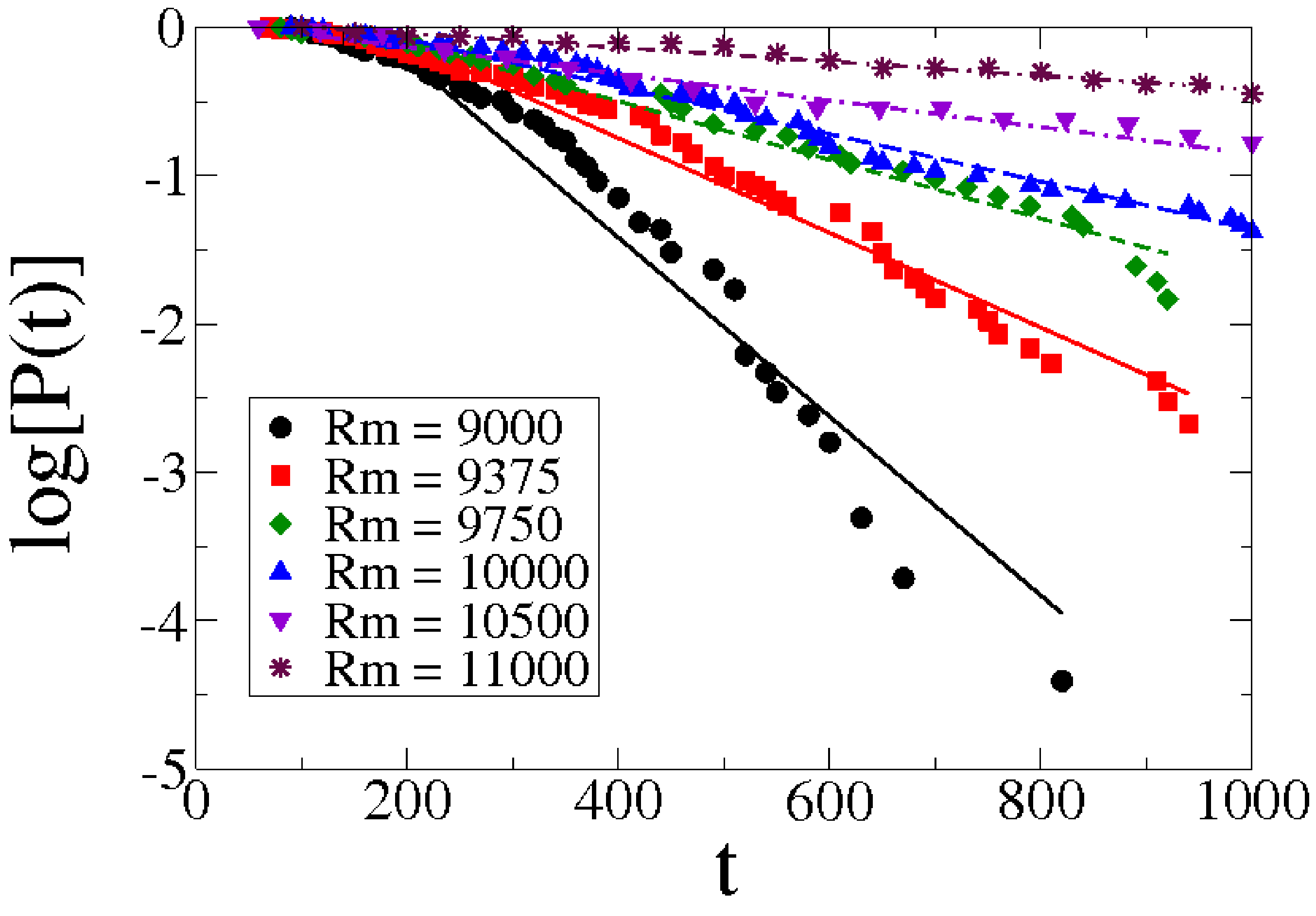
SUBCRITICAL TRANSITION TO TURBULENCE



- Laminar state is linearly stable for all Re = LU/ν
- Finite amplitude perturbations are necessary to drive the system towards a turbulent state

Source: Willis and Kerswell, Phys. Rev. Lett. 98, 014501 (2007)

KEPLERIAN SHEAR FLOWS



N_0 = total number of series (50 to 90)

N_t = number of series that are still turbulent at t

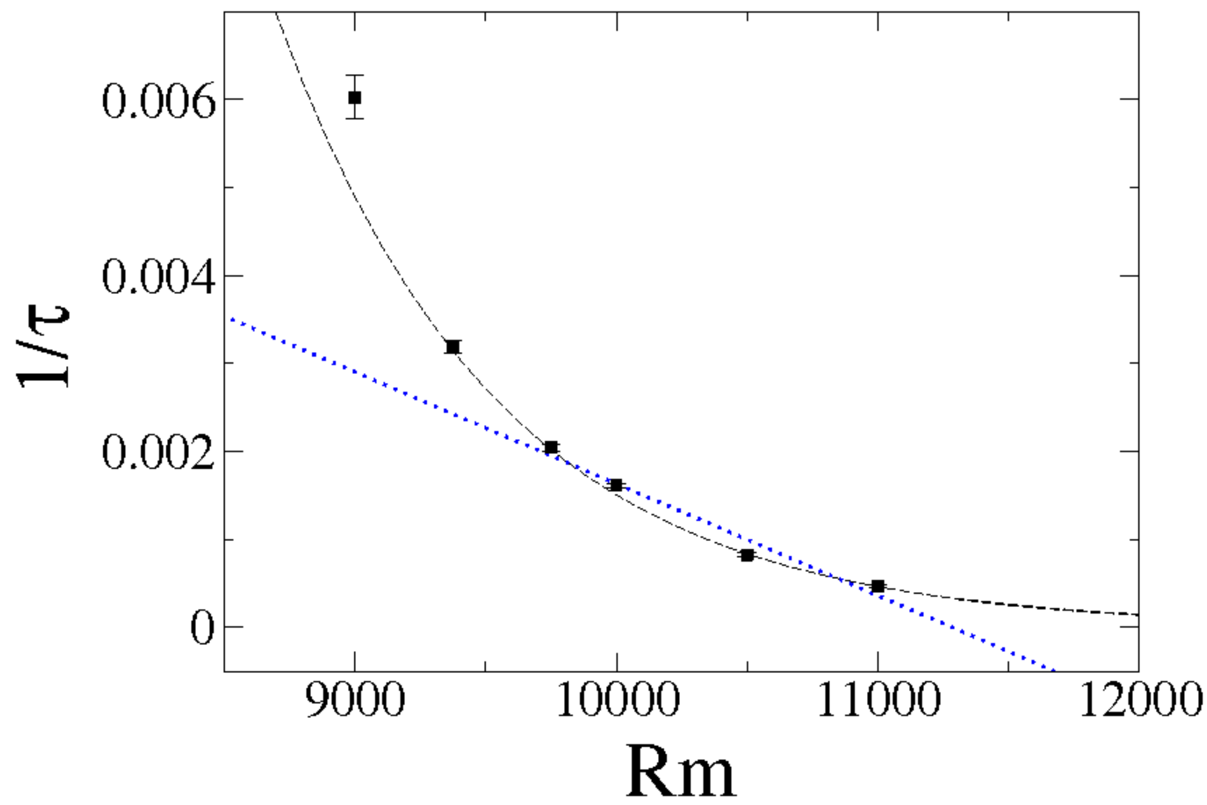
$P(t) = N_t/N_0$

Initial conditions taken from long turbulent run, 500 t.u. Appart

Over 400 runs with up to 1000 shear time units

Source: Rempel, Lesur and Proctor, Phys. Rev. Lett. 105,044501 (2010).

KEPLERIAN SHEAR FLOWS

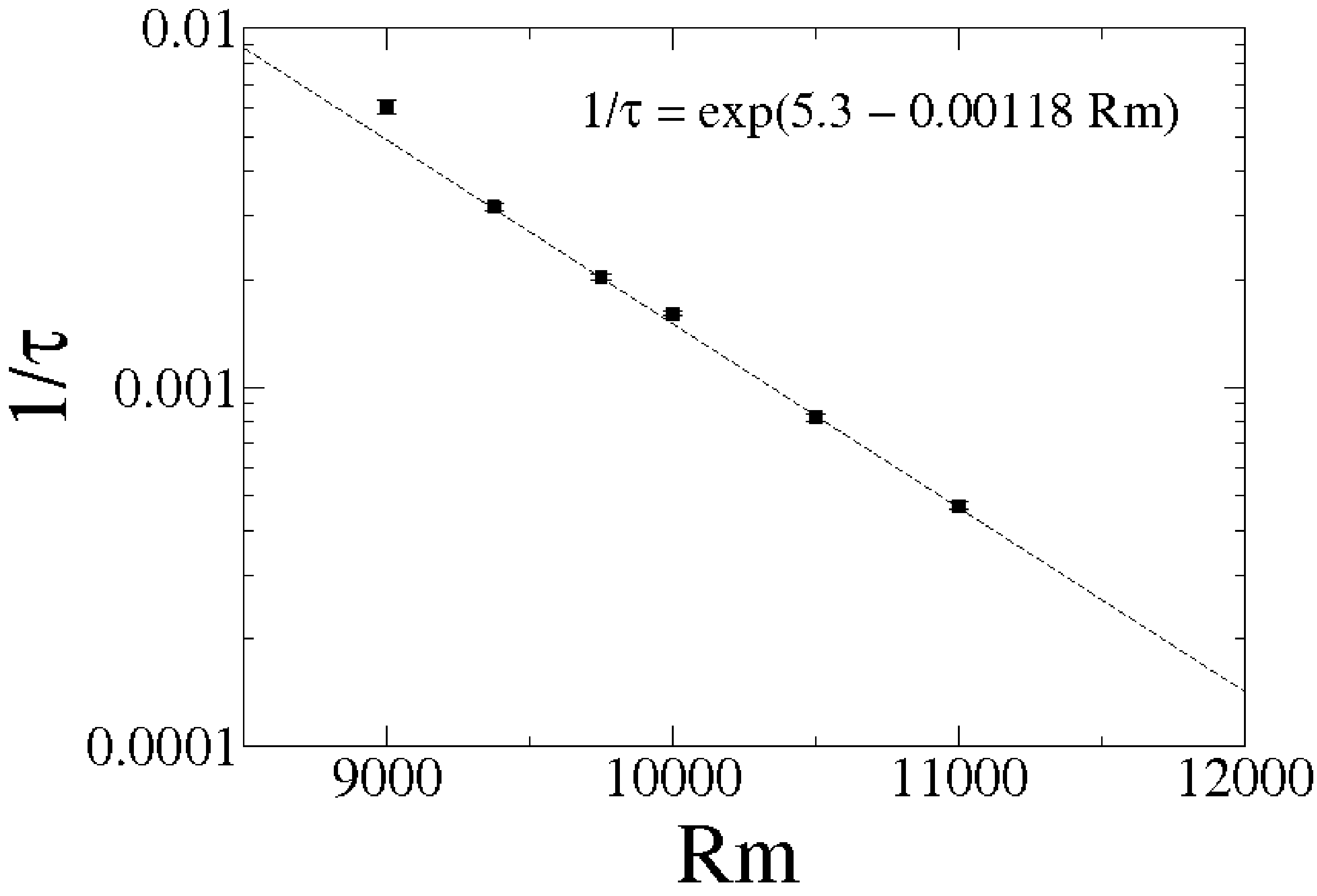


- Dashed (exponential fit): $1/\tau = \exp[5.3 - (1.18 \times 10^{-3})Rm]$
- Dotted straight line (linear fit):

  $1/\tau = 0.0144 - 1.273 \times 10^{-6} Rm \rightarrow Rmc = 11312$

Source: Rempel, Lesur and Proctor, Phys. Rev. Lett. 105,044501 (2010).

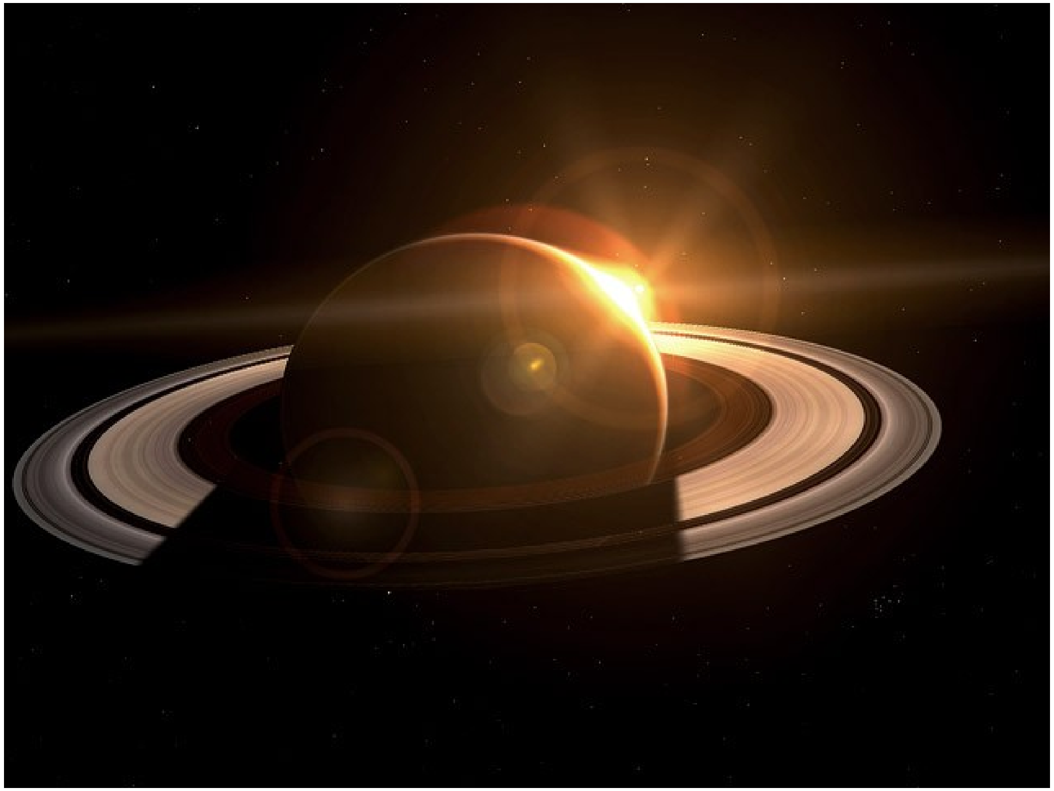
KEPLERIAN SHEAR FLOWS



Source: Rempel, Lesur and Proctor, Phys. Rev. Lett. 105,044501 (2010).

CONCLUSIONS

MHD turbulence in the incompressible Shearing box equations with zero net flux and intermediate Re and Rm is due to chaotic saddles and, therefore, is not self-sustained.



Final attractor is laminar, and there is no accretion in the long term.

Decay time follows a supertransient Law.

In practice, the decay time could be huge due to high values of Re and Rm.

MRI

Two radially neighbouring fluid elements behave as two mass points connected by a massless spring, due to the Lorenz force

If m1 moves is displaced inwardly, it will have higher angular velocity

Magnetic tension will pull m1 back and drag m0 forward

m1 looses angular momentum L and falls to an inner orbit, corresponding to smaller L

m0 acquires more L and moves to a higher orbit.

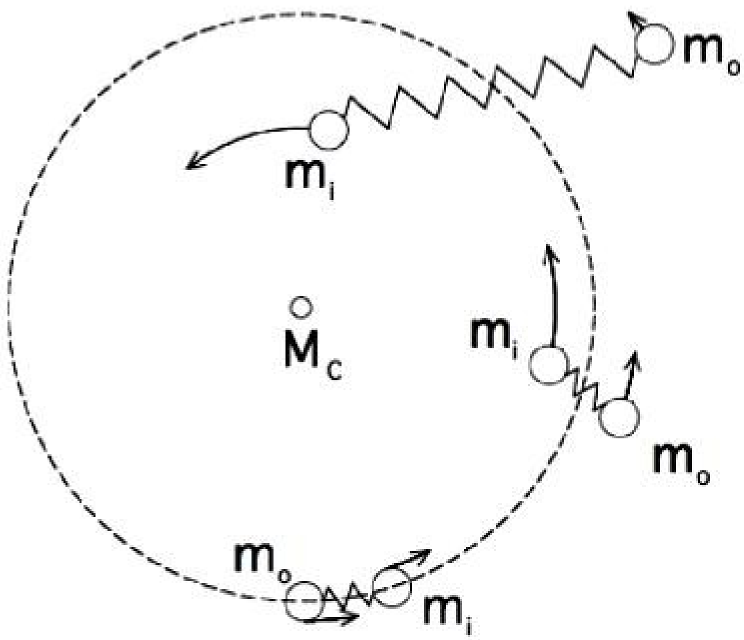
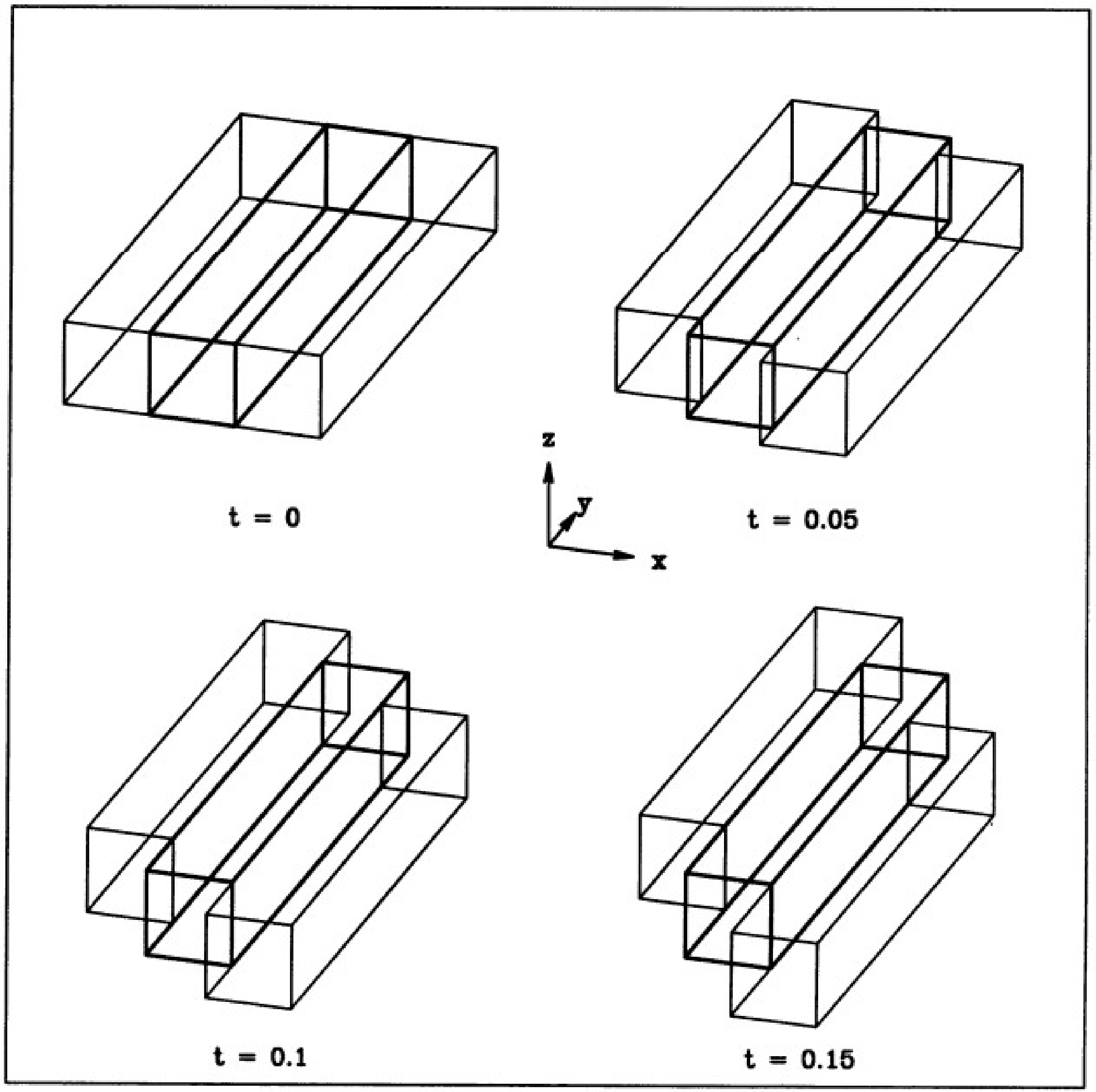


Image credit: http://www.scholarpedia.org/article/Magnetorotational_instability

MODELING KEPLERIAN SHEAR FLOWS



- $f(x,y,z)=f(x+L_x,y-SL_x t,z)$
- $f(x,y,z)=f(x,y+L_y,z)$
- $f(x,y,z)=f(x,y,z+L_z)$
- $v_y(x,y,z)=v_y(x+L_x,y-SL_x t,z) + SL_x$
- Strictly periodic box in:

  $t = nL_y / (SL_x)$, n = 1,2,3,...

Source: Hawley, Gammie, and Balbus, ApJ 440, 742-763 (1995).

DECAY TIME

- Let R be a region of the phase space containing a chaotic saddle and no attractors
- Let N_0 be a large number of initial conditions in R
- Most trajectories will eventually leave R
- Let N_t be the number of trajectories still in R after t time units

Source: Hsu, Ott, and Grebogi, Phys. Lett. A 127, 199 (1988).

DECAY TIME

- Due to the saddle points present in the chaotic saddle, N_t decays exponentially with time t at the rate $1/\tau$, where the escape time τ is defined as:

$N_t = N_0 \exp[-(t-t_0)/\tau]$ (as $N_0 \rightarrow \infty$ and $t \rightarrow \infty$)

$1/\tau = \log(N_0/N_t)/(t-t_0)$

Source: Hsu, Ott, and Grebogi, Phys. Lett. A 127, 199 (1988).

KEPLERIAN SHEAR FLOWS

- Recall that for chaotic saddles:

  $1/\tau = \log(N_0/N_t)/(t-t_0)$ (as $N_0 \to \infty$ and $t \to \infty$)

  $= \log(P(t)^{-1})/(t-t_0)$

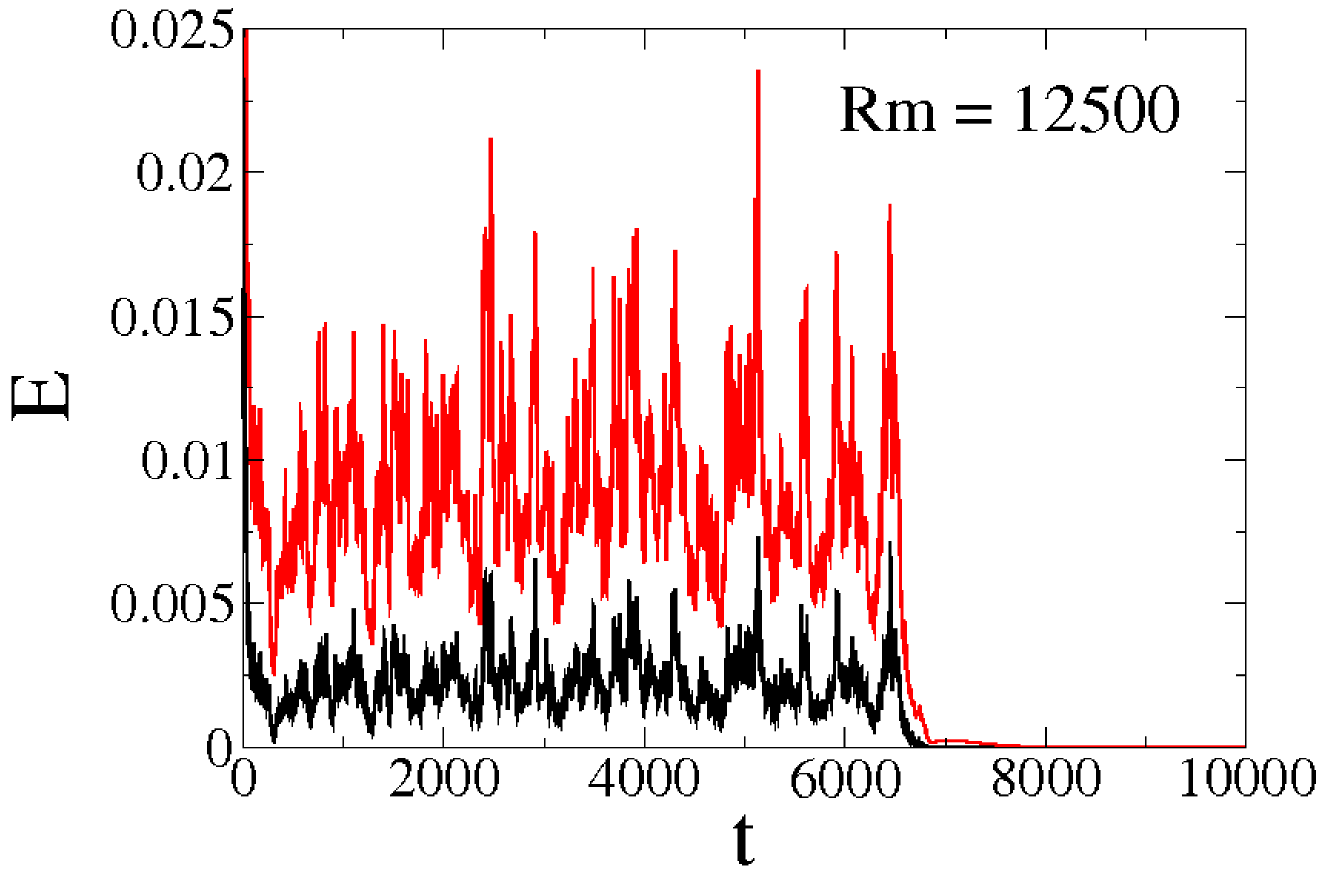
  $= -\log(P(t))/[t-t_0]$

  $1/\tau \sim [\log(1)-\log(P(t))]/[t-t_0]$

- $1/\tau$ is obtained from the slopes of the fitted lines

Source: Hsu, Ott, and Grebogi, Phys. Lett. A 127, 199 (1988).

KEPLERIAN SHEAR FLOWS



- Transient time series of magnetic (red) and kinetic (black) energies for Re=3125 and Rm=12500.

Source: Rempel, Lesur and Proctor, Phys. Rev. Lett. 105,044501 (2010).

KEPLERIAN SHEAR FLOWS

Statistical data from Fig. 5