Properties of MHD Turbulence

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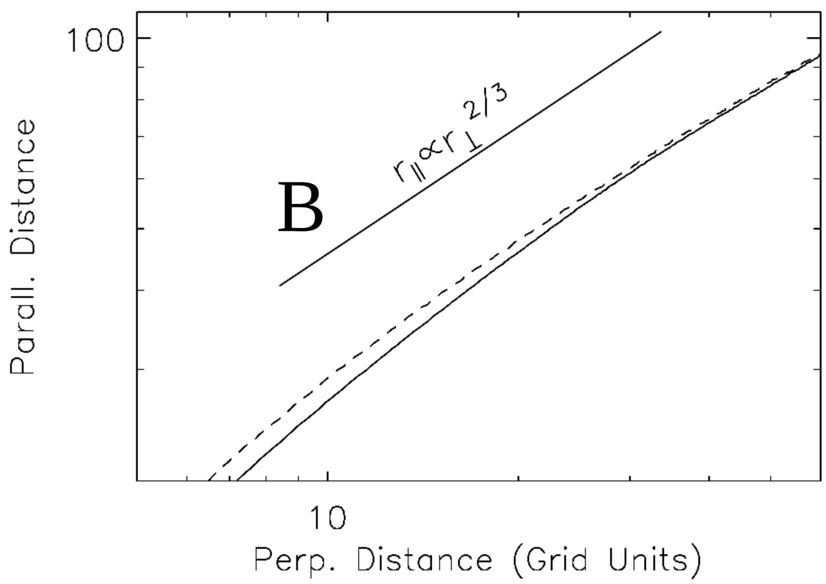
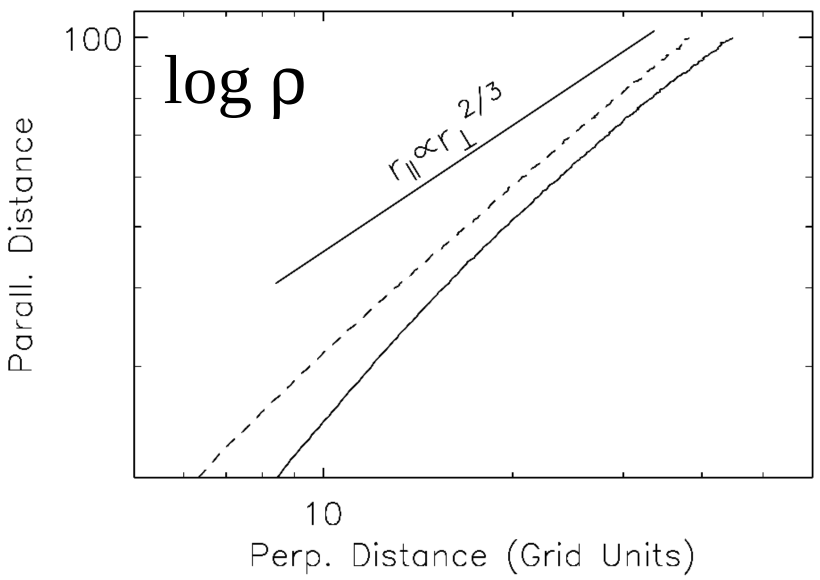
Basic properties of MHD turbulence

(zero total cross-helicity, zero total magnetic helicity,

infinitely strong mean field, strong turbulence)

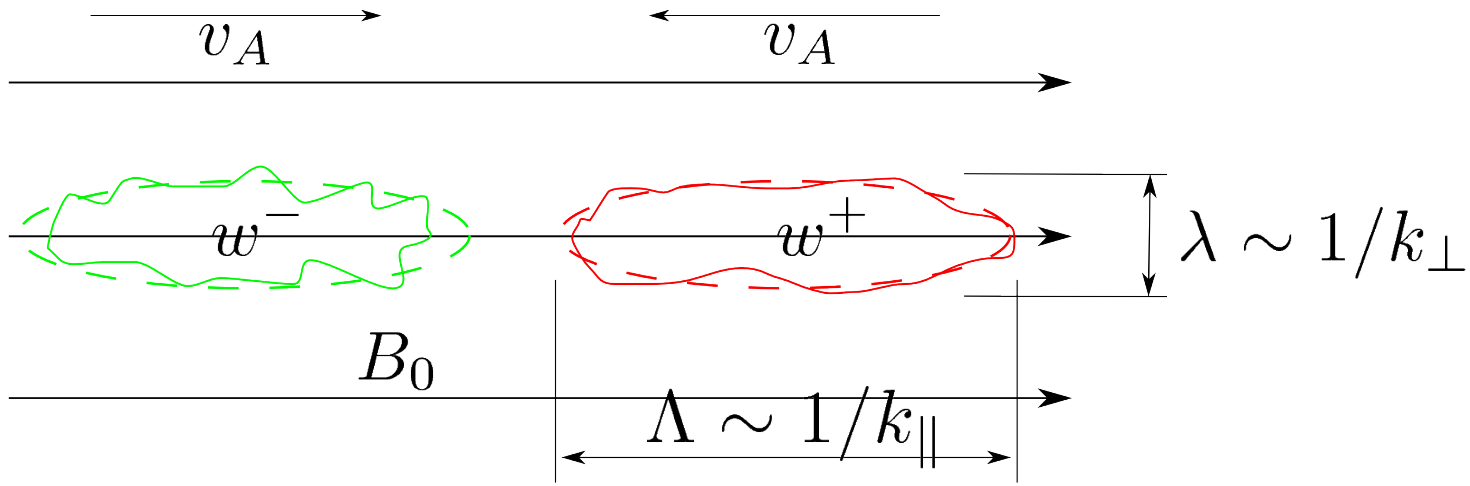
It's not just an abstract theory – signatures of *Alfvenic* turbulence are also present in supersonic low-beta MHD turbulence, i.e. **ISM turbulence**, see, e.g. anisotropy of B and log density in Mach-10 MHD:





AB et al ApJL (2005)

Basic properties of MHD turbulence



$k_\parallel$ is conserved, $k_\perp$ is increasing

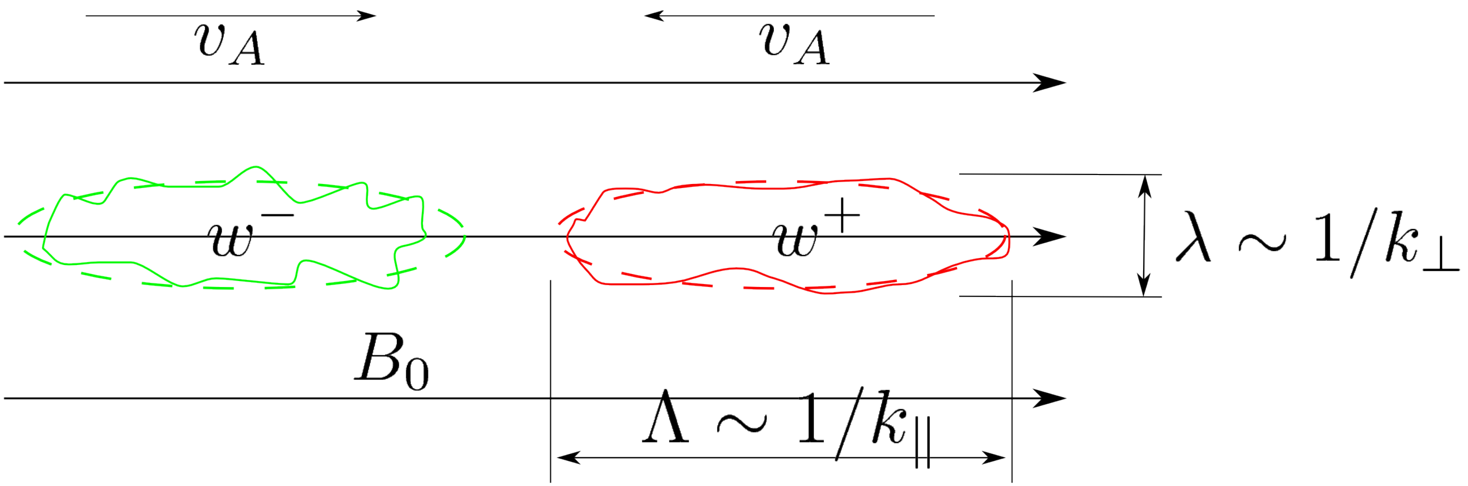
$$k_\parallel \ll k_\perp$$

$$\partial_t \delta \mathbf{w}^\pm \mp (\mathbf{v_A} \cdot \nabla)\delta \mathbf{w}^\pm + \hat{S}(\delta \mathbf{w}^\mp \cdot \nabla)\delta \mathbf{w}^\pm = 0$$

$\not{k}_\parallel, k_\perp$

could be split in two equations

Basic properties of MHD turbulence



$$k_\parallel \ll k_\perp$$

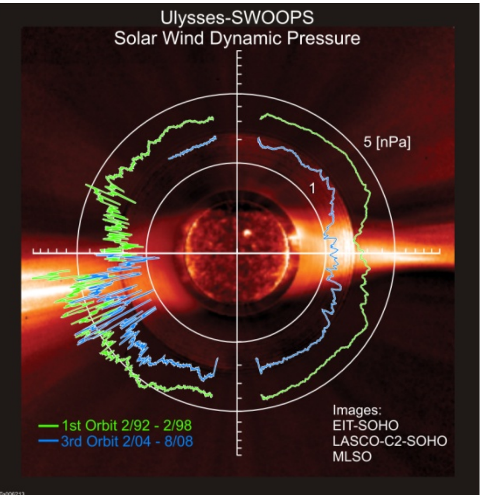
Alfvenic dynamics (a.k.a. "reduced MHD") has essential nonlinearity:

$$\partial_t \mathbf{w}_\perp^\pm \mp (\mathbf{v_A} \cdot \nabla_\parallel)\mathbf{w}_\perp^\pm + \hat{S}(\mathbf{w}_\perp^\mp \cdot \nabla_\perp)\mathbf{w}_\perp^\pm = 0$$

Slow mode is passively mixed:

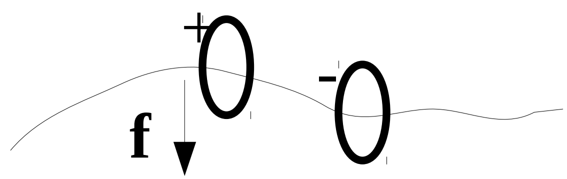
$$\partial_t w_\parallel^\pm \mp (\mathbf{v_A} \cdot \nabla_\parallel) w_\parallel^\pm + \hat{S}(\mathbf{w}_\perp^\mp \cdot \nabla_\perp) w_\parallel^\pm = 0$$

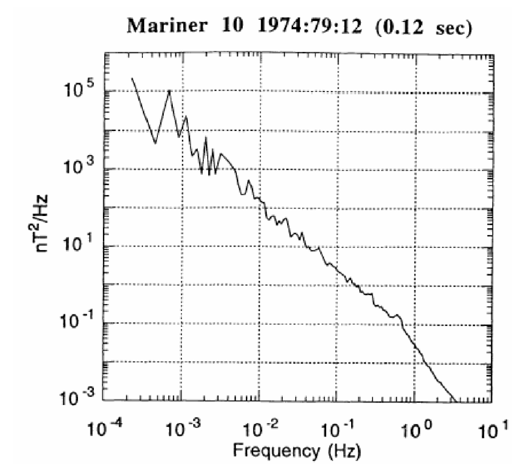
Alfvenic turbulence

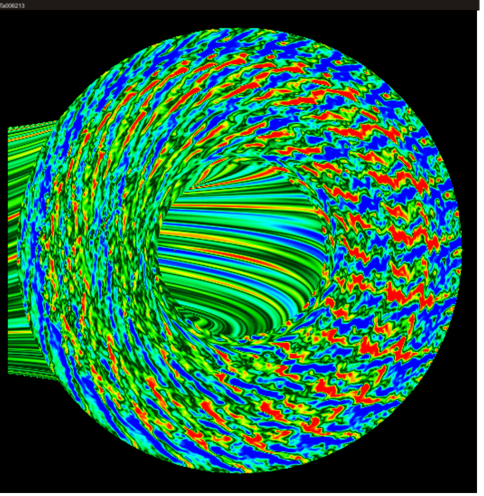


Reduced (Alfvenic) MHD could be derived for weakly collisional plasmas as Alfven mode does not require pressure support.

Density fluctuations in the solar wind are much smaller than you would expect from transonic flow-- it is mostly an Alfvenic flow.







$$\partial_t \mathbf{w}_\perp^\pm \mp (\mathbf{v_A} \cdot \nabla_\parallel)\mathbf{w}_\perp^\pm + \hat{S}(\mathbf{w}_\perp^\mp \cdot \nabla_\perp)\mathbf{w}_\perp^\pm = 0$$

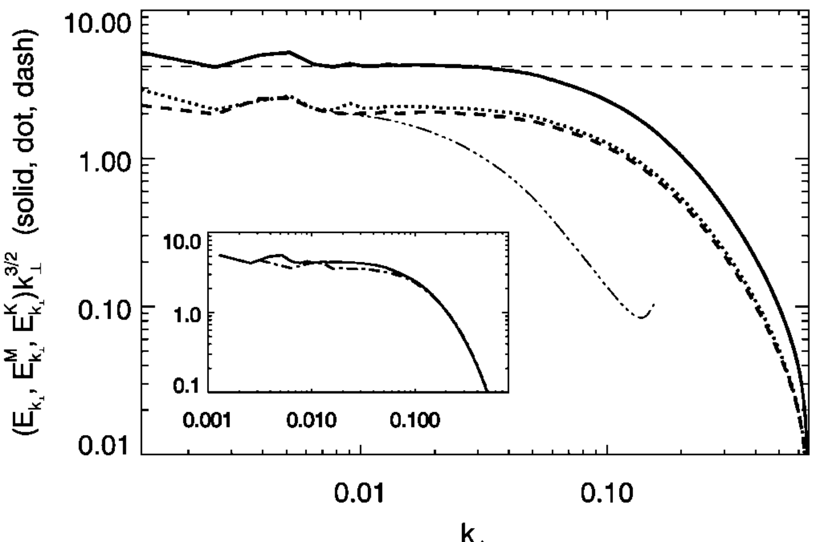
A universality is possible:

$$w \to wA, \quad \lambda \to \lambda B, \quad t \to tB/A, \quad \Lambda \to \Lambda B/A$$

Energy spectral slopes: -5/3 or -3/2?

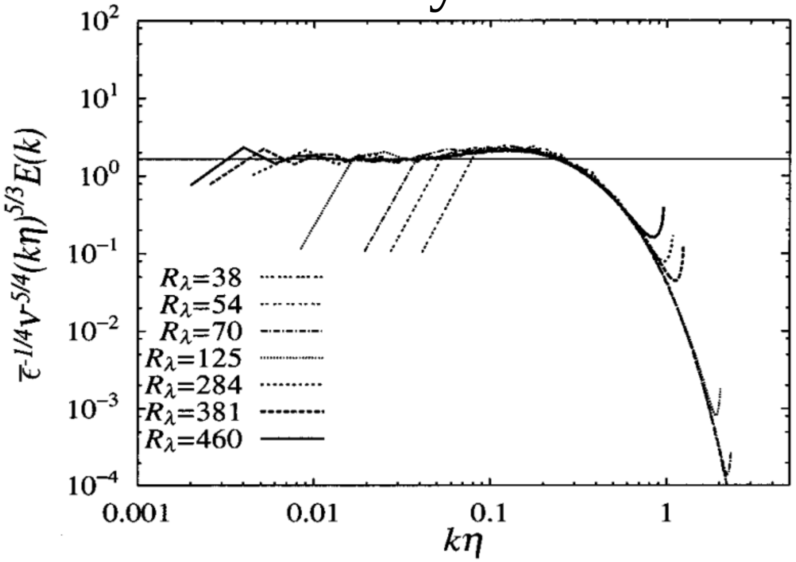
Goldreich-Sridhar model predicts -5/3 but
shallower slopes are often observed in simulations.

MHD, strong mean field



Mueller & Grappin 2005

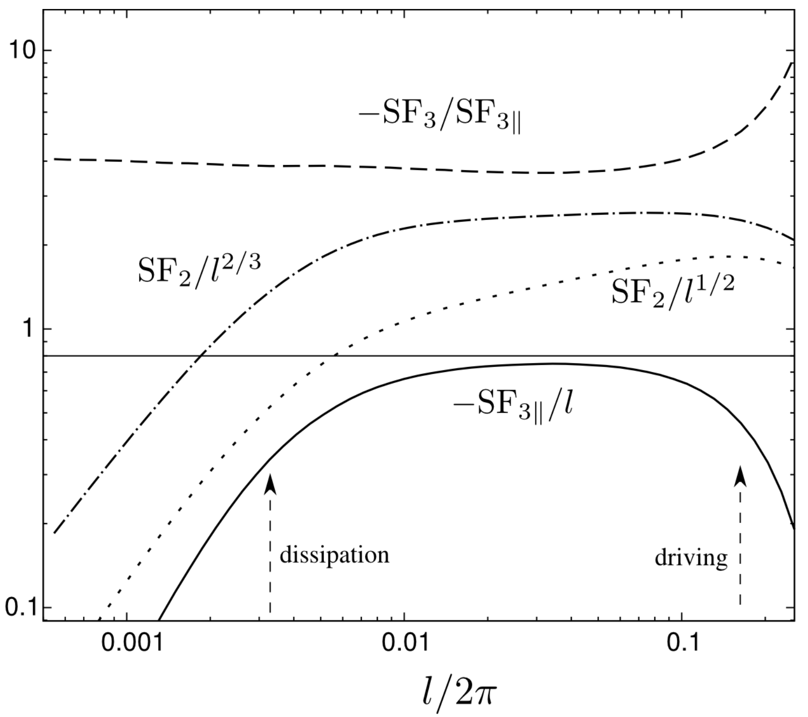
Hydro



Gotoh et al 2002

Structure functions

Hydro

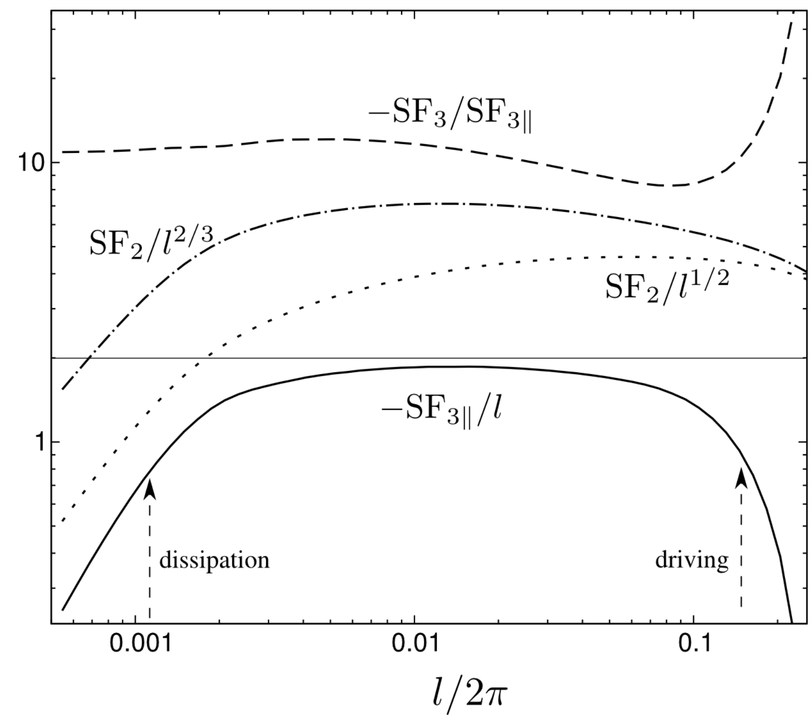


$$SF_{3\|} = < \delta v_{\|}^3 >$$

$$SF_3 = < |\delta v|^3 >$$

$$SF_2 = < |\delta v|^2 >$$

MHD



$$SF_{3\|} = < \delta w_{\|}^{\pm} (\delta w^{\mp})^2 >$$

$$SF_3 = < |\delta w^{\pm}|^3 >$$

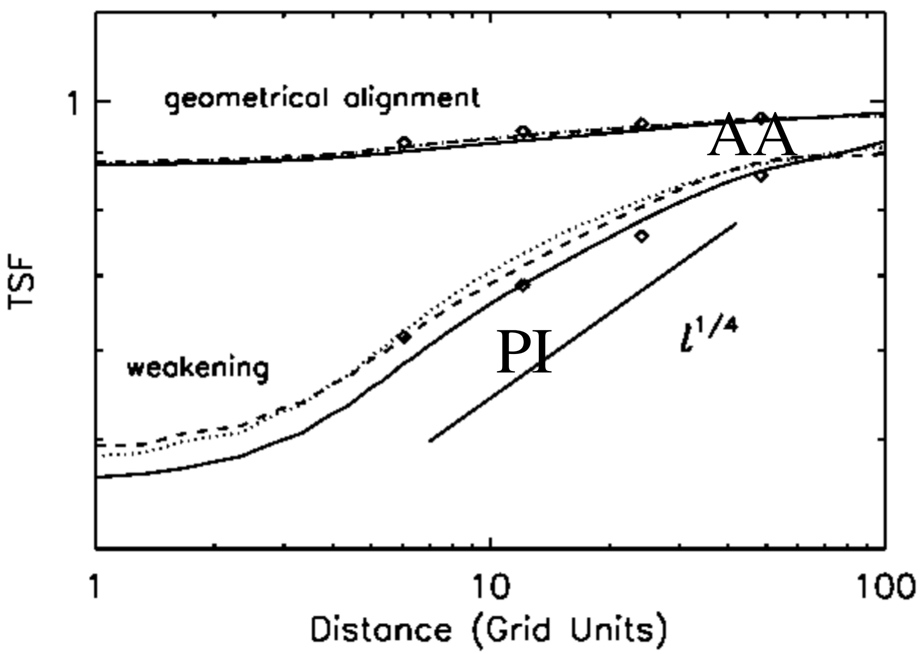
$$SF_2 = < |\delta w^{\pm}|^2 >$$

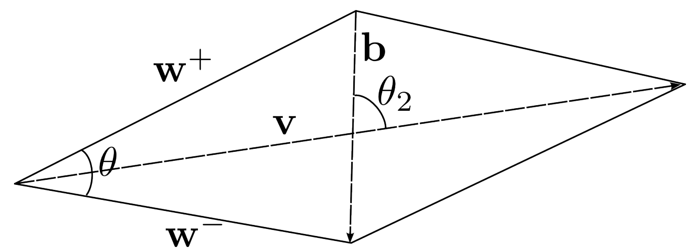
MHD scaling is not obvious – more rigorous approach is needed

“Dynamic alignment”

Boldyrev (2005) proposed “dynamic alignment” which will weaken the interaction and produce -3/2 slope (could be -13/9~-1.44 though).

AB & Lazarian (2005):





$$\mathrm{AA} = \langle |\sin\theta| \rangle$$

$$\mathrm{PI} = \langle |\delta\mathbf{w}^+ \times \delta\mathbf{w}^-| \rangle / \langle |\delta w^+ \delta w^-| \rangle$$

$$\mathrm{DA} = \langle |\delta\mathbf{v} \times \delta\mathbf{b}| \rangle / \langle |\delta v \delta b| \rangle$$

note that $\delta\mathbf{w}^+ \times \delta\mathbf{w}^- = -2\delta\mathbf{v} \times \delta\mathbf{b}$

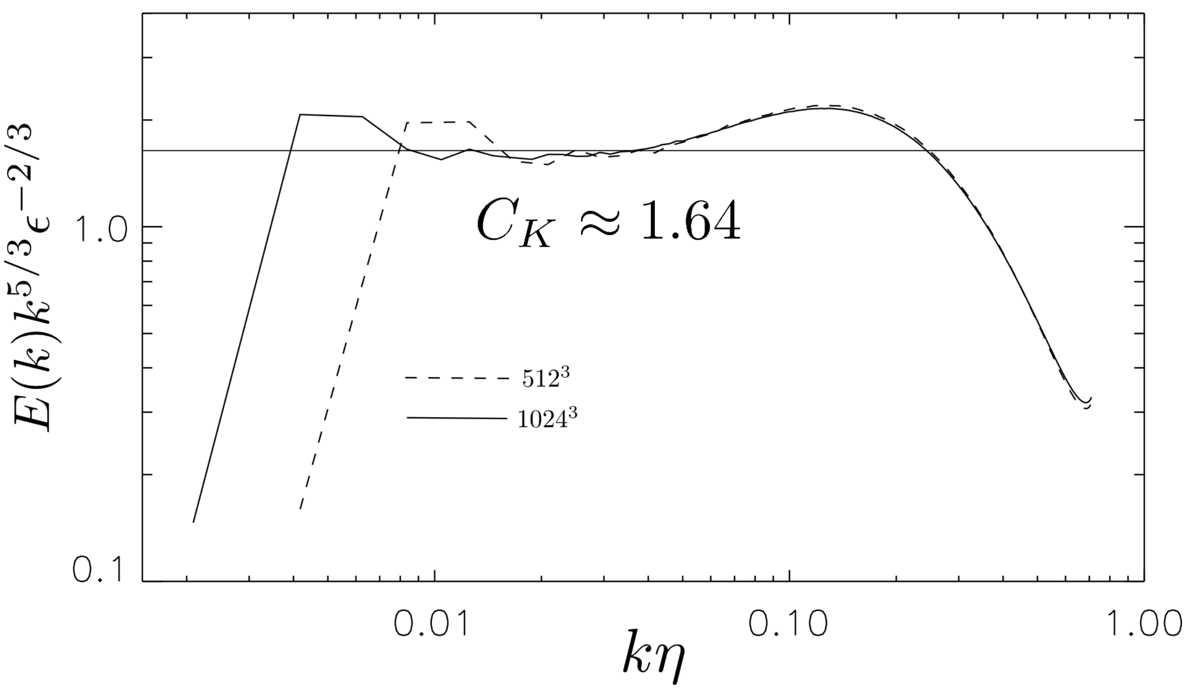
Price of the question:

- Almost a factor of 20 in power on the dissipation scales in the ISM
- Highly unusual aligned fields on small scales (v and B aligned within an angle of 10^{-3})
- Kolmogorov constant and dissipation rates vary by a factor of 2-3

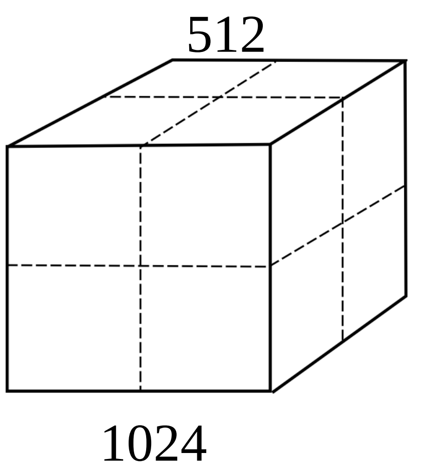
Resolution study

Each simulation produces a 2-parametric set of solutions, which is associated with symmetries of the numerical equations.

Hydro:



Larger resolution could mean larger scales

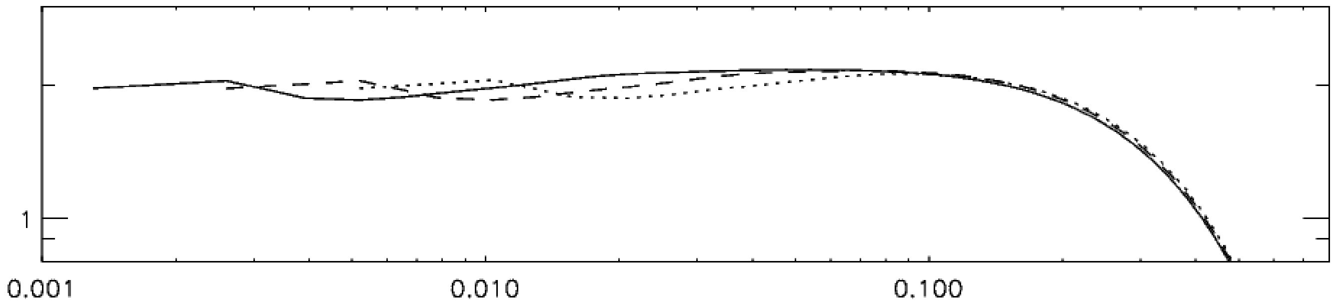


Resolution study is a **rigorous** way to claim a correspondence or a lack of it with a particular universal scaling

MHD convergence study: $576^3, 1152^3, 2304^3$

for -5/3 model

$P(k)k^{5/3}\epsilon^{-2/3}$

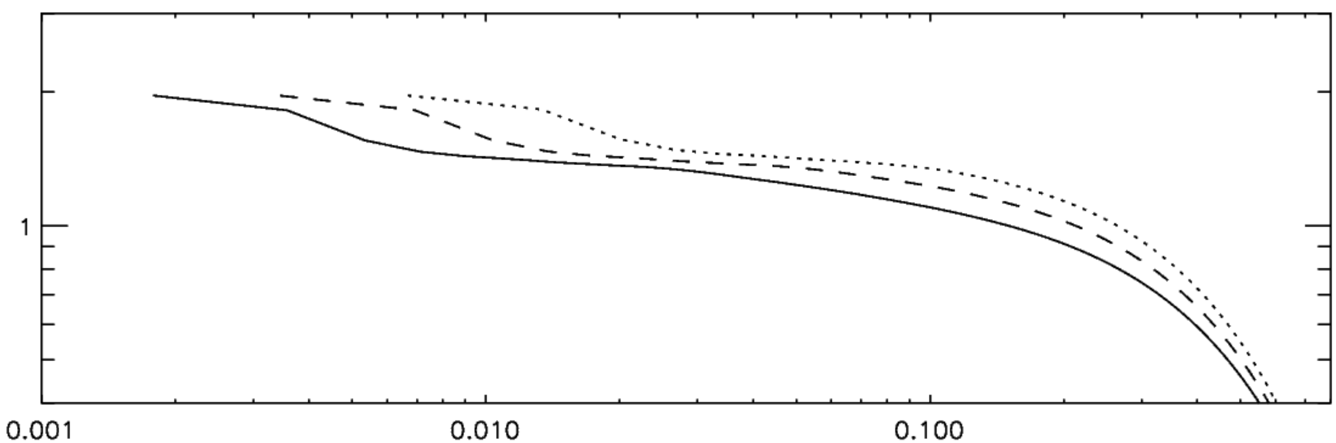


$k\eta$

Best convergence for slope -1.7, same as for hydro.

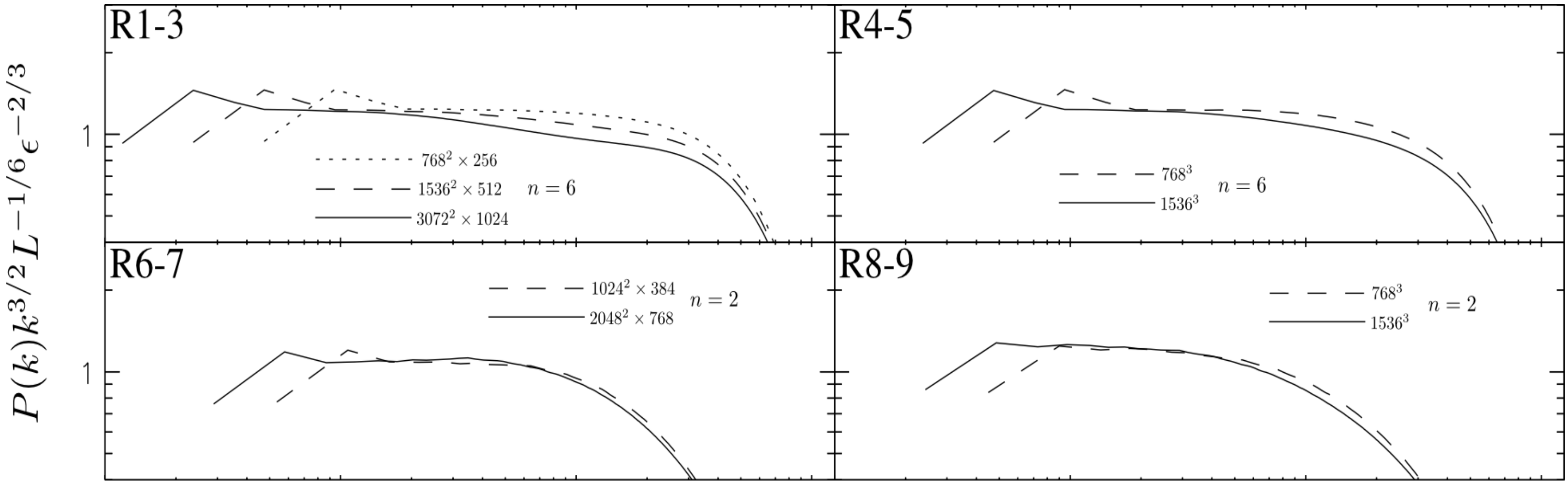
for -3/2 model

$P(k)k^{3/2}L^{-1/6}\epsilon^{-2/3}$

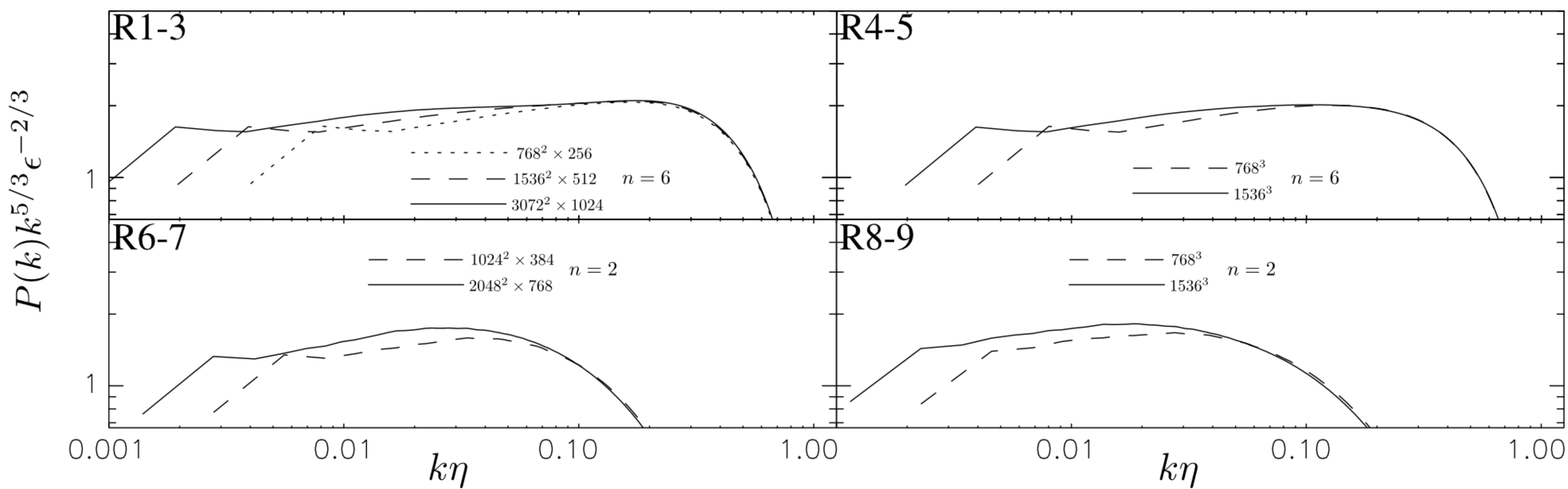


$k\eta$

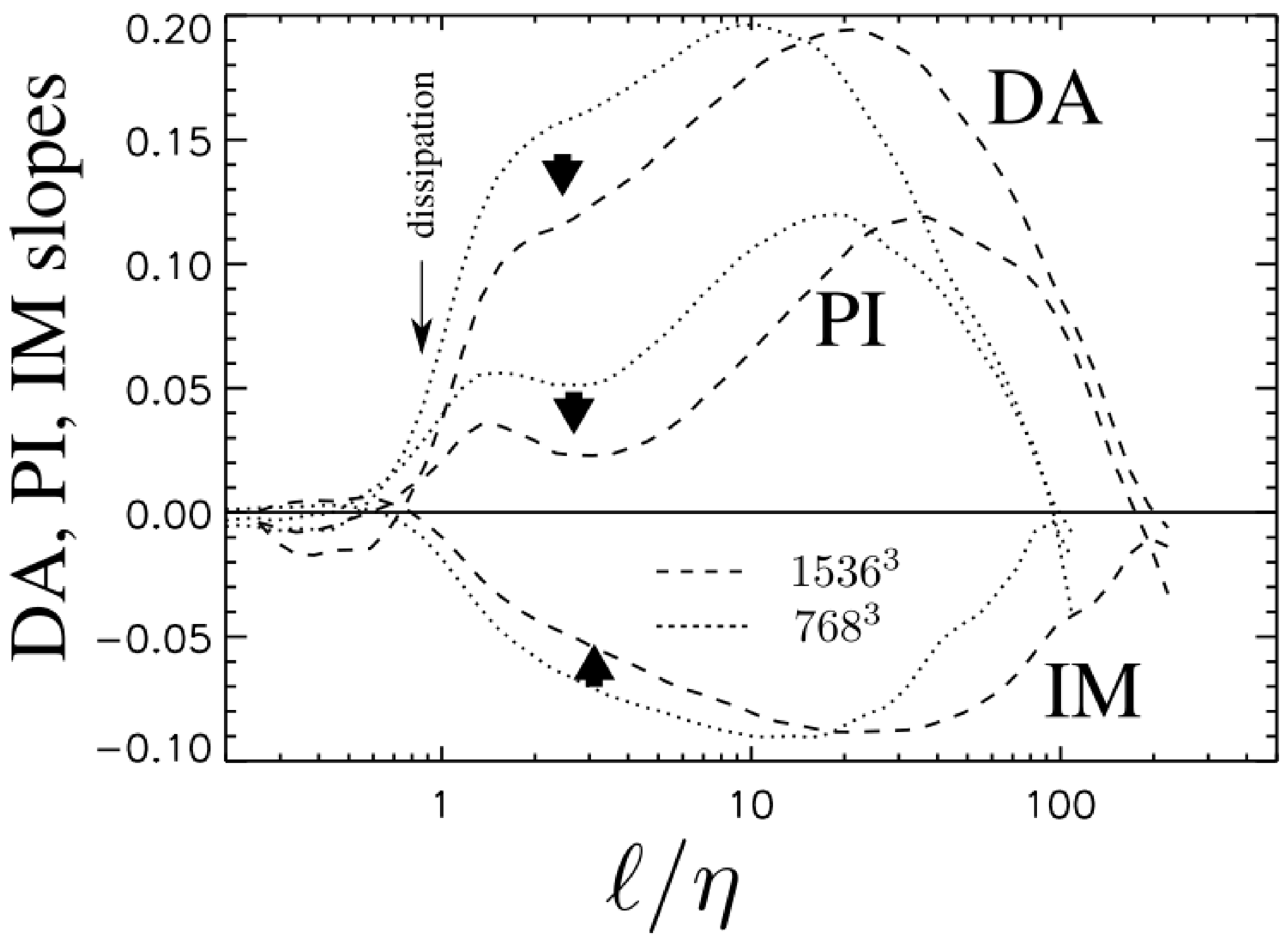
Convergence study for -3/2 model



Convergence study for -5/3 model



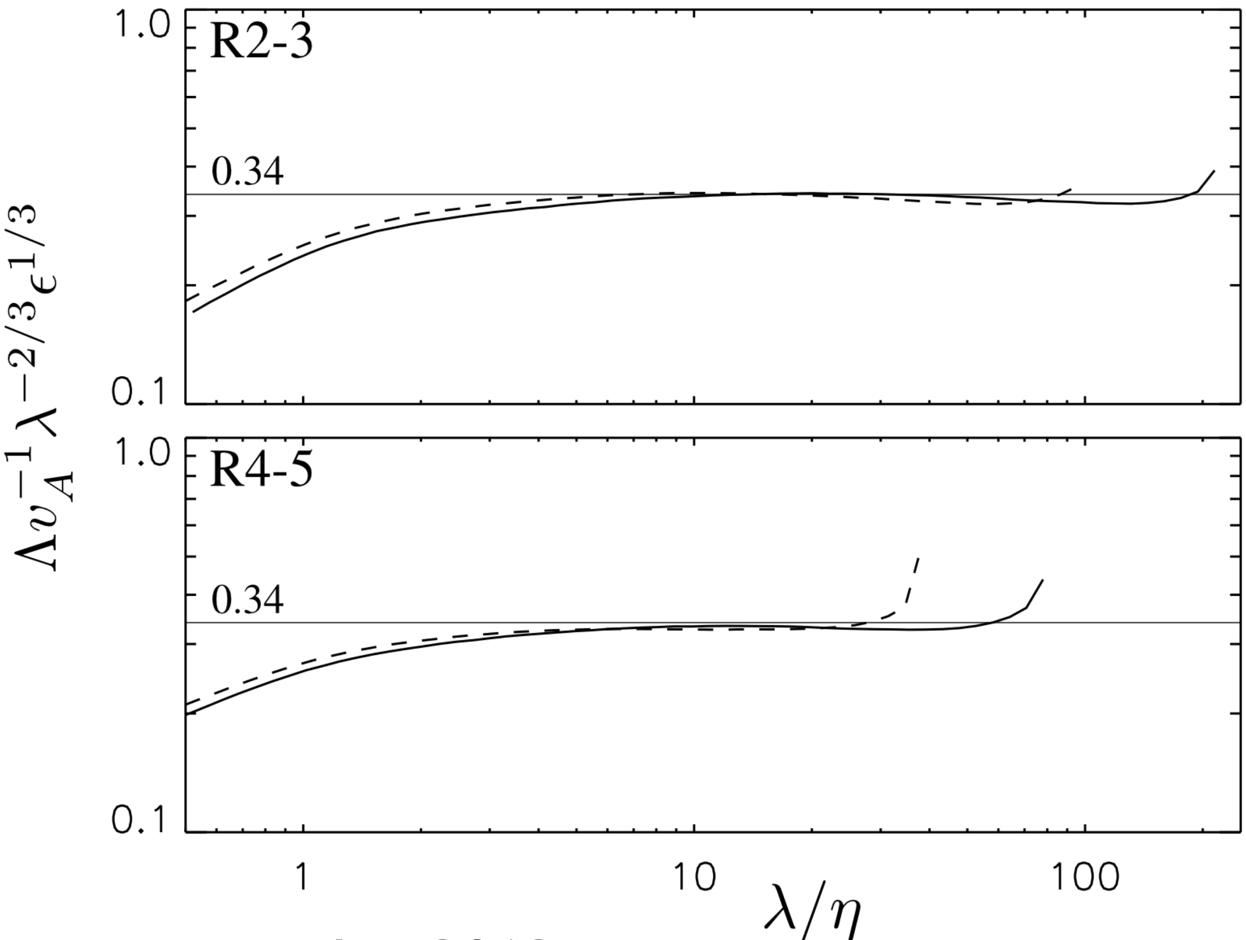
Alignment shows no convergence:



A particular universal slope ¼ is predicted in the alignment model, it is inconsistent with numerics.

Universal anisotropy

$$\Lambda_{\|} = C_A v_A \lambda_\perp^{2/3} \epsilon^{-1/3}$$



$3072^2 \times 1024$

$1536^2 \times 512$

1536^3

768^3

AB, 2012

$C_A C_K^{1/2} \approx 0.62$

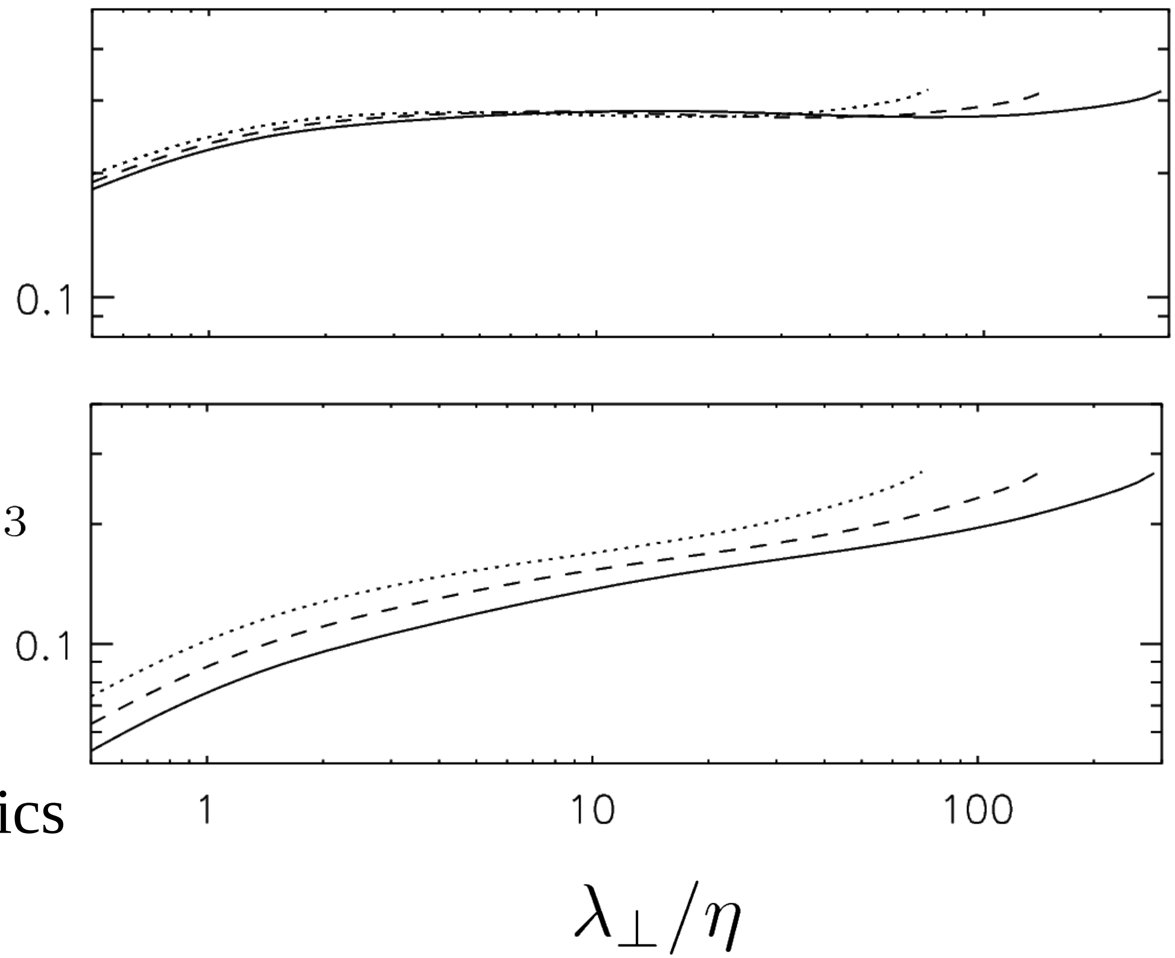
Universal anisotropy

$$\boxed{\Lambda_{\|}} = C_A \boxed{v_A} \lambda_\perp^{2/3} \epsilon^{-1/3}$$

2304^3

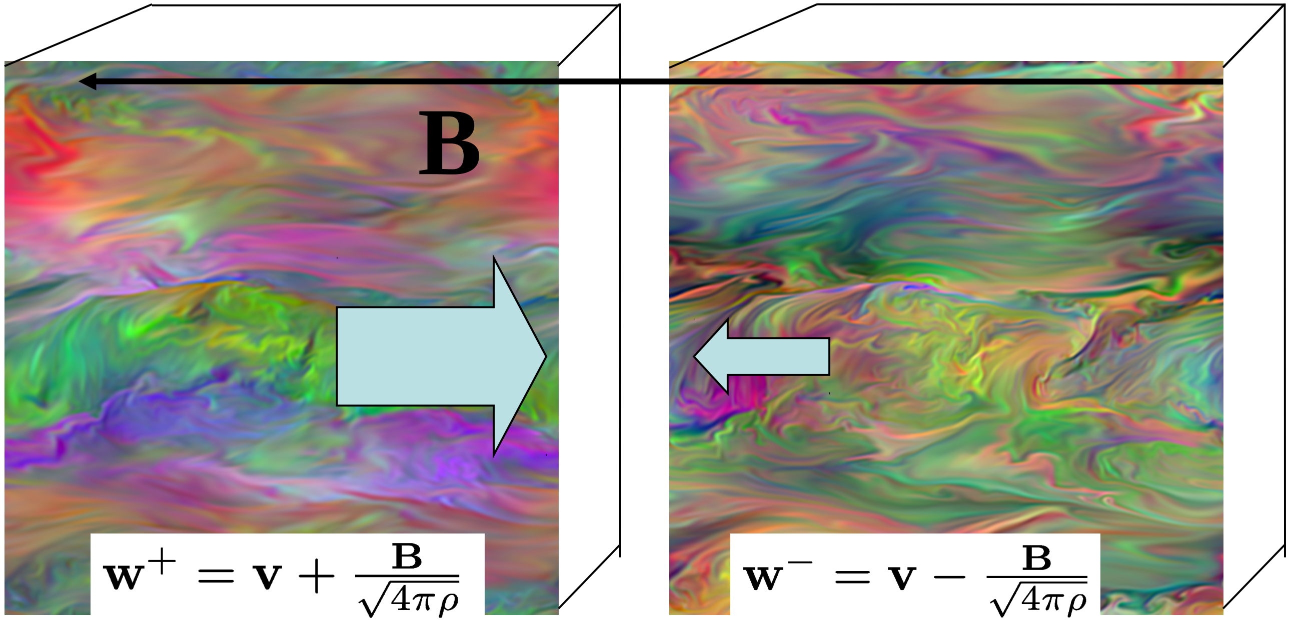
$$\Lambda_{\|} v_A^{-1} \lambda_\perp^{-2/3} \epsilon^{1/3}$$

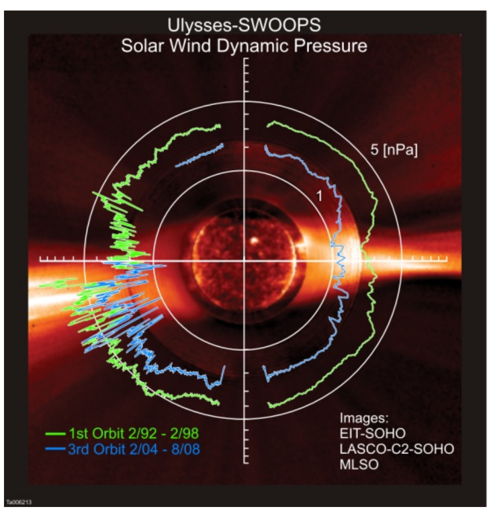
$$\Lambda_{\|} v_A^{-1} \lambda_\perp^{-1/2} L^{-1/6} \epsilon^{1/3}$$

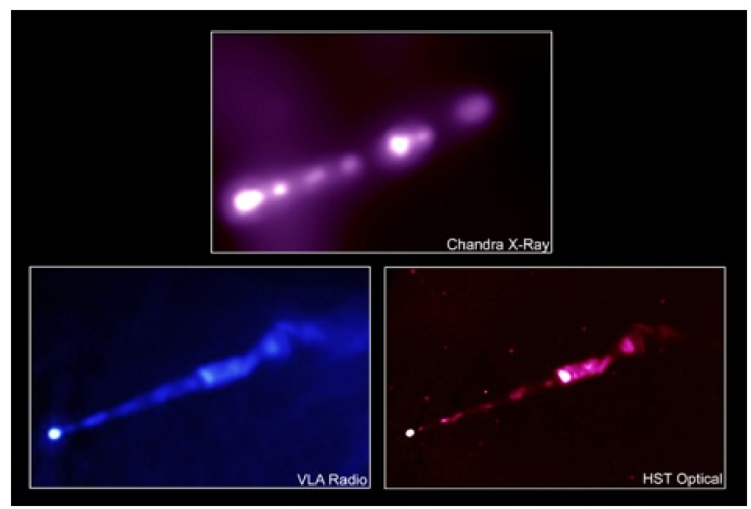


Boldyrev anisotropy--
inconsistent with numerics

Imbalanced turbulence







Basic Measurements

$$(w^{\pm})^2 = E(1 \pm \sigma_C)/2 \text{ – Elsasser energy}$$

$$\tau^{\pm} \text{ – nonlinear timescale}$$

$$\epsilon^{\pm} \text{ – dissipation rate}$$

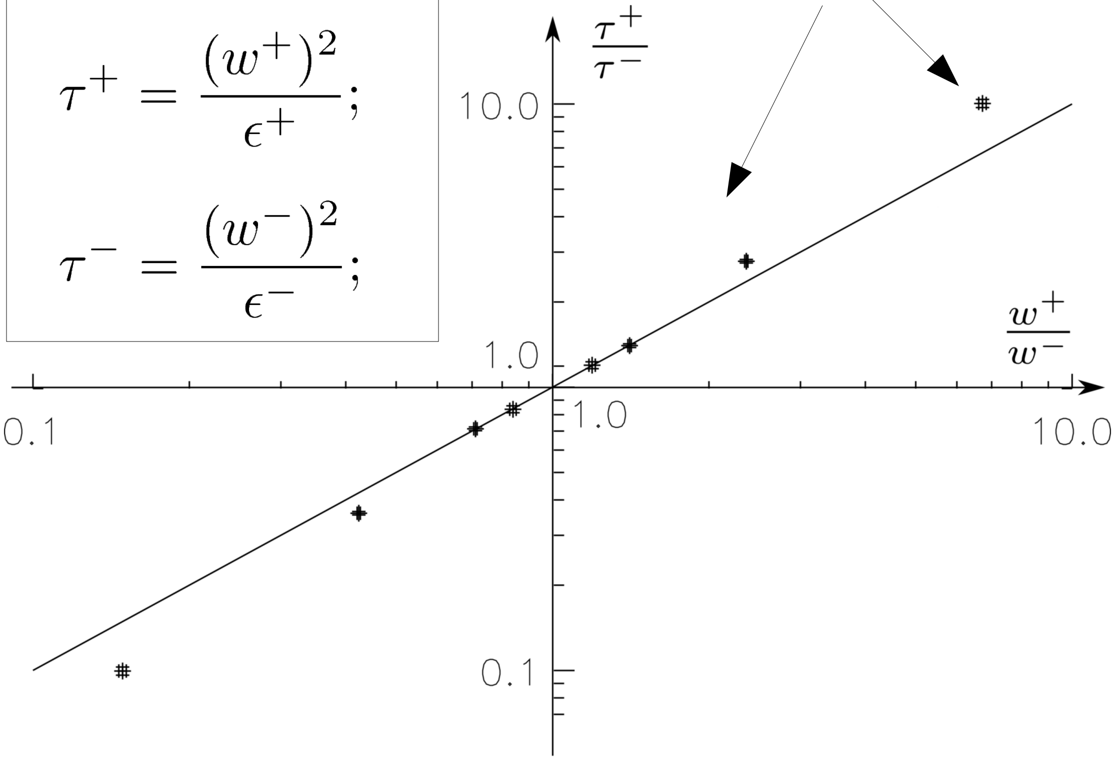
Energy cascade

obtained in simulations

$$\tau^{+} = \frac{(w^{+})^2}{\epsilon^{+}};$$

$$\tau^{-} = \frac{(w^{-})^2}{\epsilon^{-}};$$

Data from 1024x1536^2 simulations



A model of strong imbalanced turbulence

n

Old critical balance (causality) $\Lambda^- = v_A \left(\frac{w^+(\lambda_1)}{\lambda_1}\right)^{-1}$; $\left(\frac{\Lambda^+}{\lambda_1}\right)^{-1} = \frac{w^+(\lambda_2)}{v_A}$ New critical balance (field wandering)

energy — shear rate

$$\epsilon^- = \frac{(w^-(\lambda_1))^2 w^+(\lambda_1)}{\lambda_1}$$

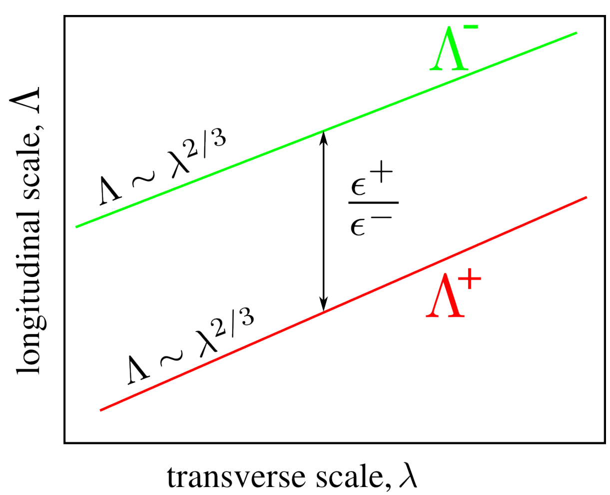
Strong cascading of weak wave

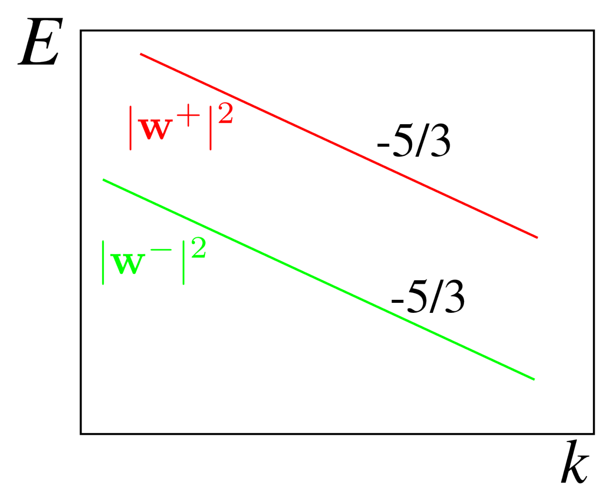
$$\epsilon^+ = \frac{(w^+(\lambda_2))^2 w^-(\lambda_1)}{\lambda_1} \cdot \frac{w^-(\lambda_1)\Lambda^-}{v_A \lambda_1} \cdot f(\lambda_1/\lambda_2)$$

weakening factor

Weak cascading of strong wave

Asymptotic power-law solutions:

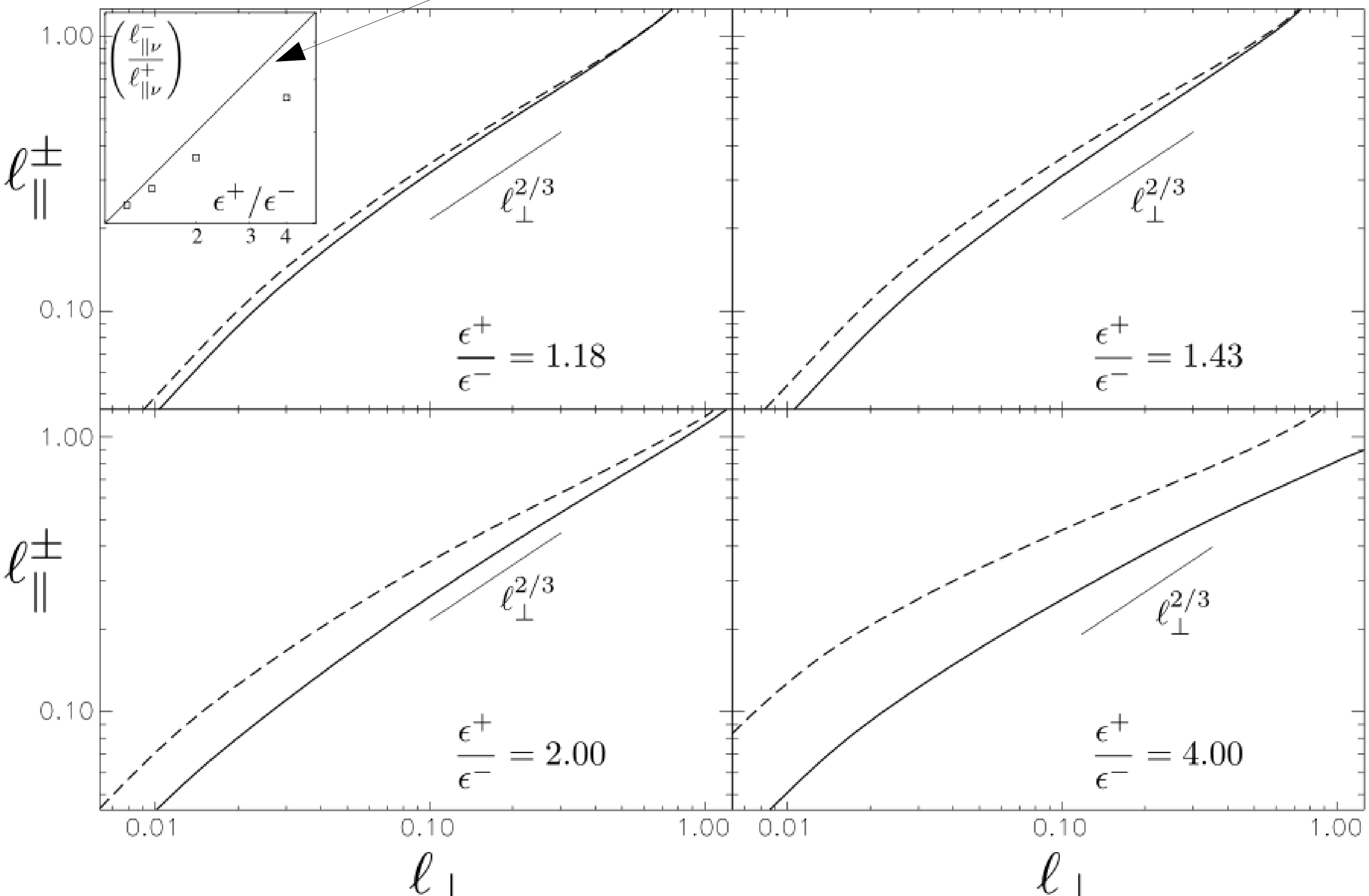




AB & Lazarian, 2008

Anisotropy in the imbalanced turbulence

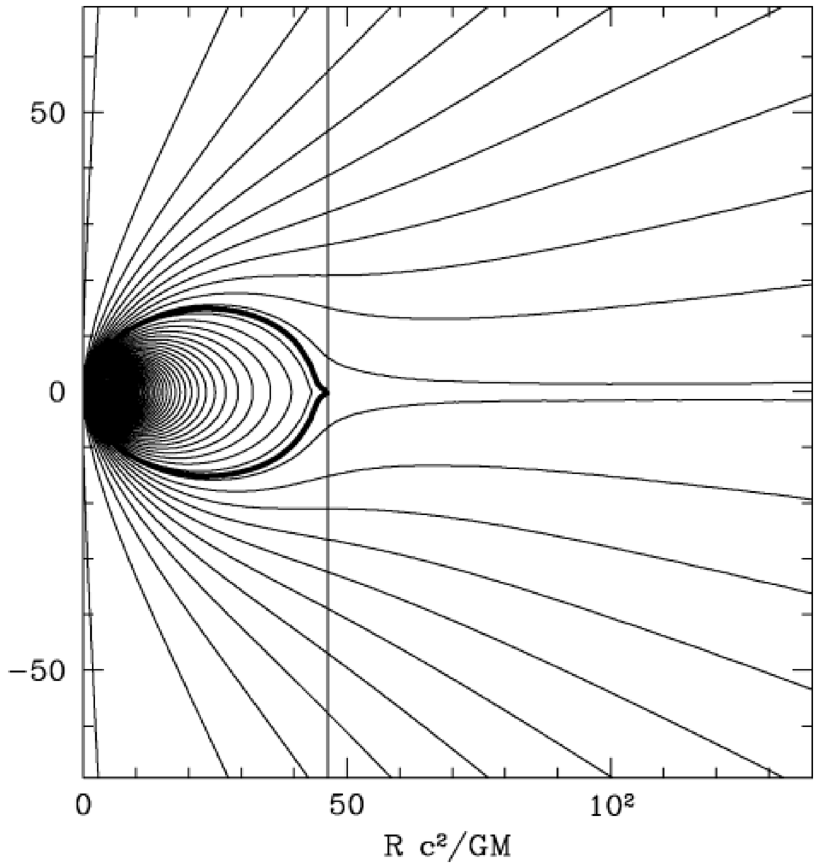
AB & Lazarian 2008



AB, 2012

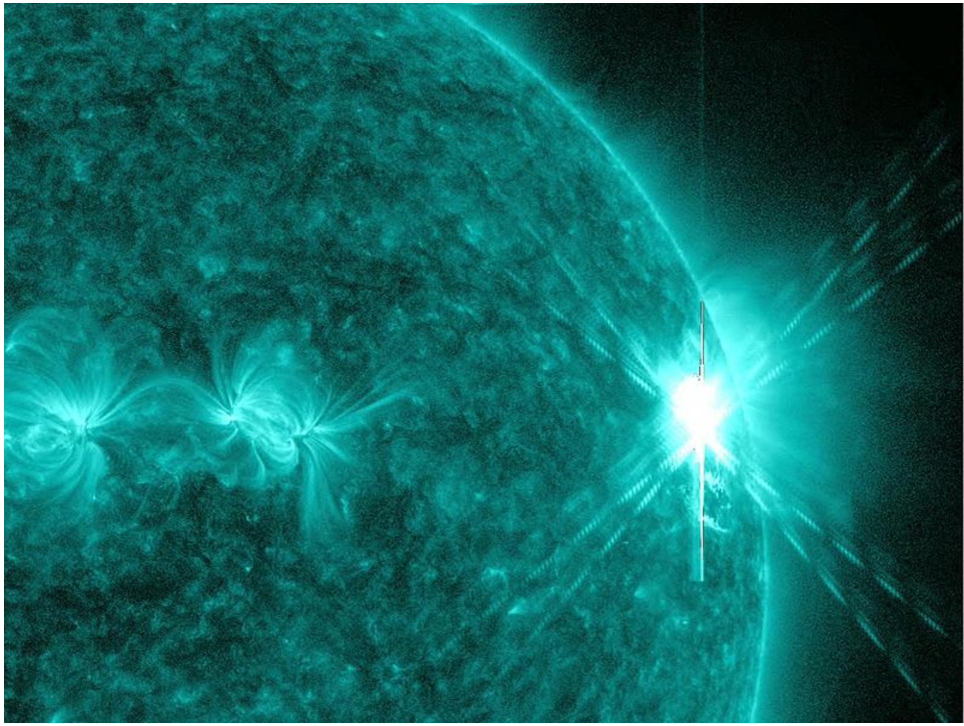
Dissipation in current layers

Pulsar magnetosphere



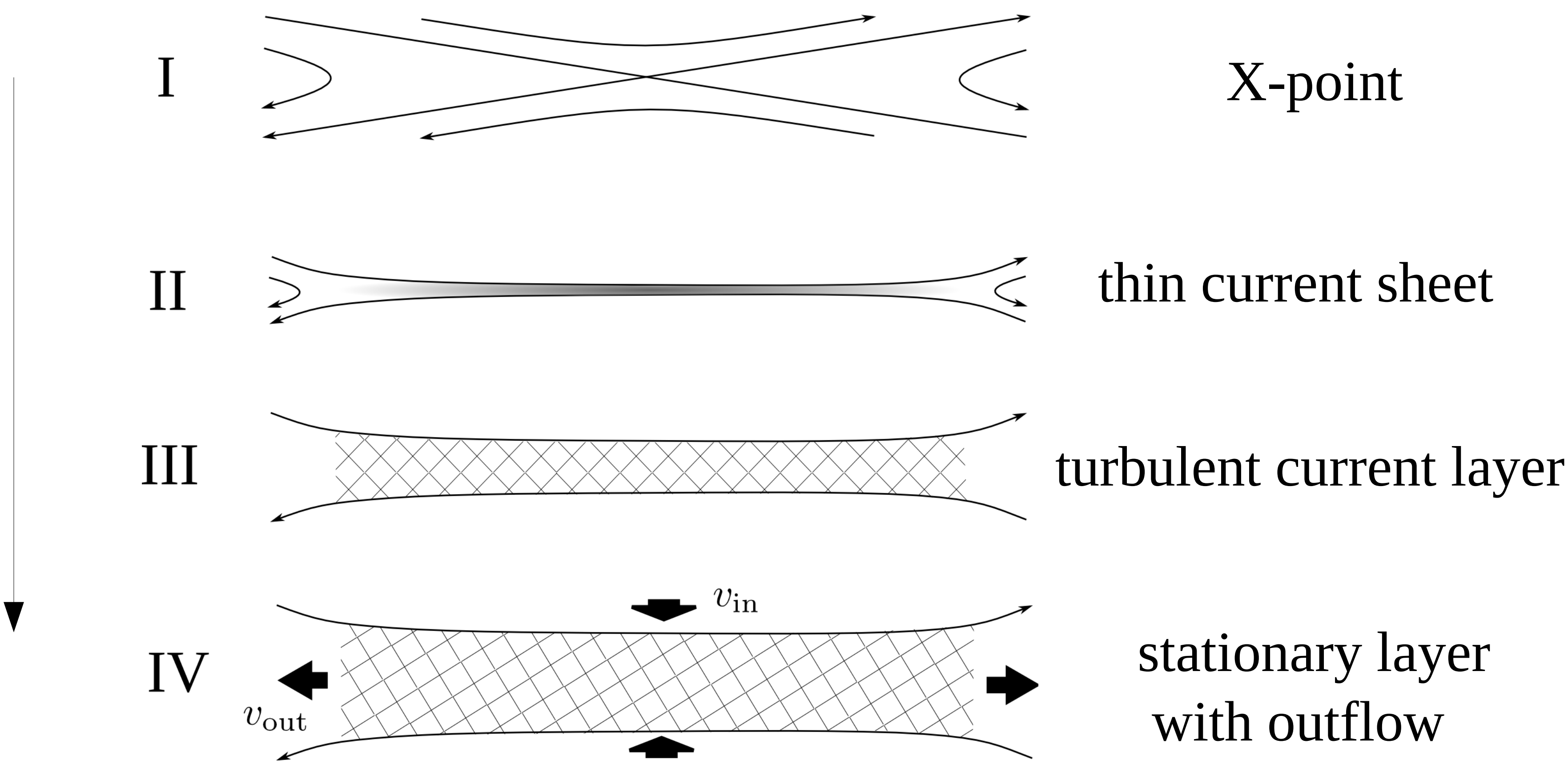
McKinney 2006

Solar flare

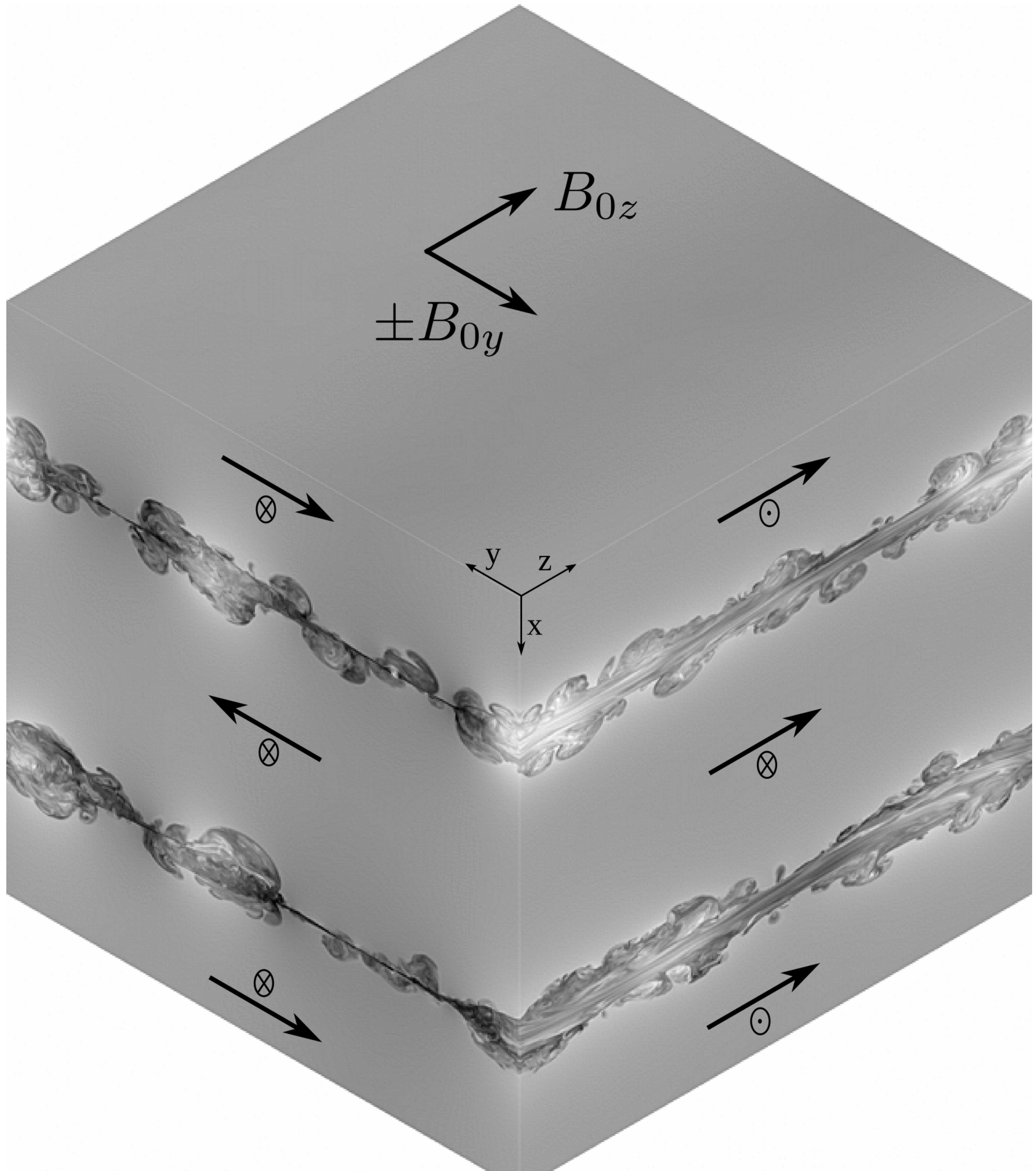


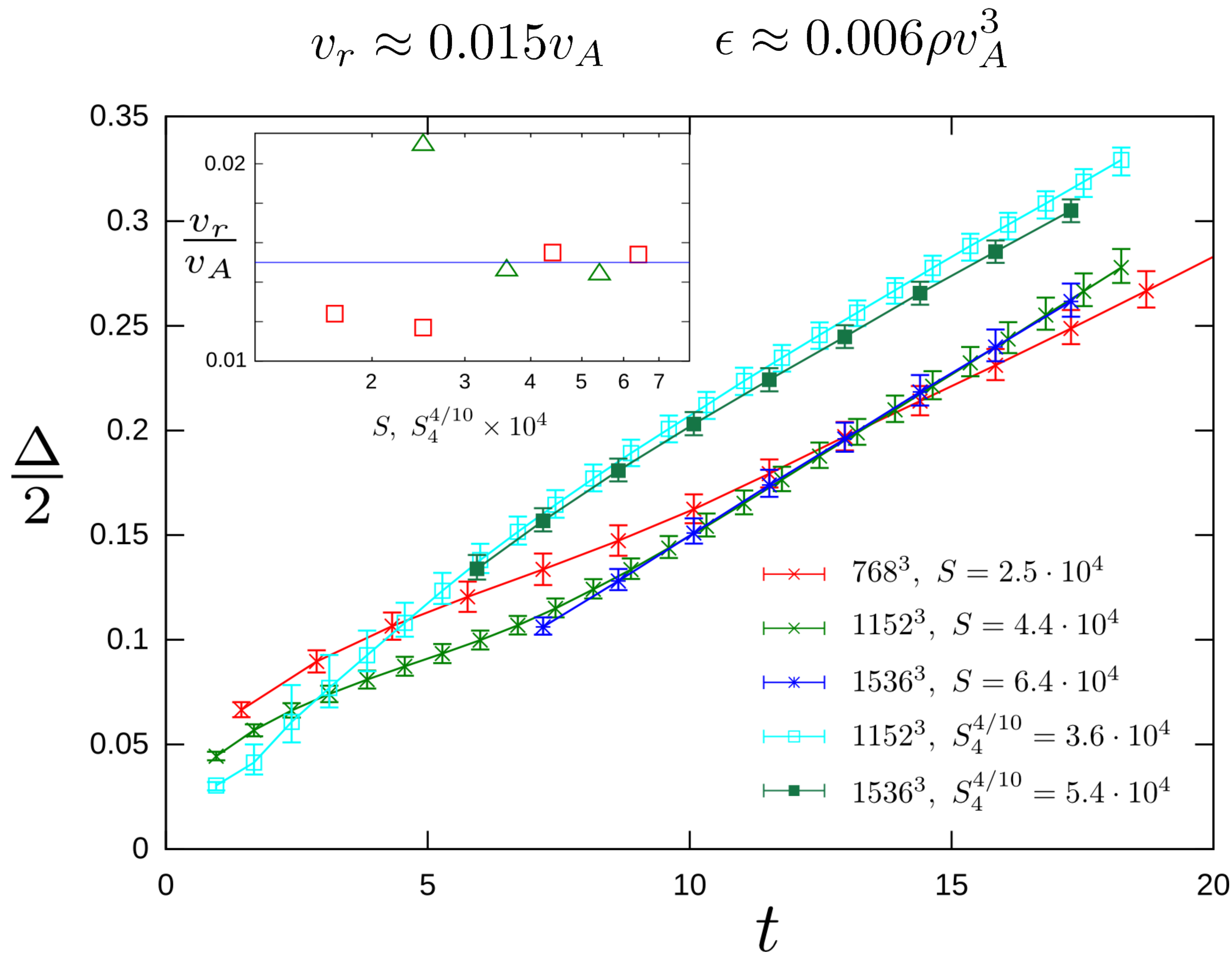
SOHO

Thin current sheets are unstable to tearing mode if $S = v_A L/\eta > 10^4.$



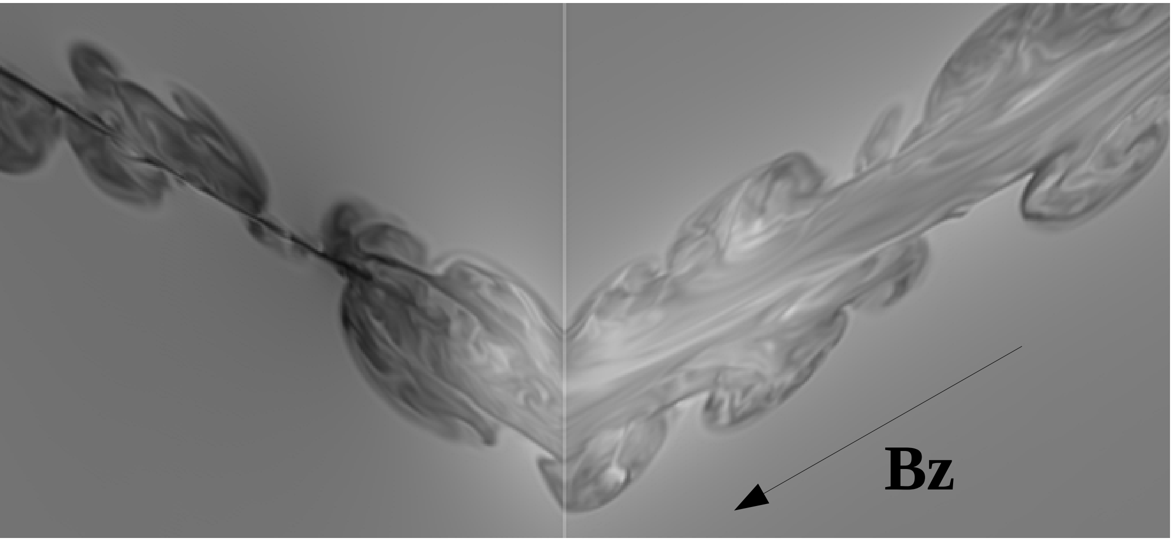
3D MHD simulations of nonlinear current sheet instability or MHD turbulence driven by current

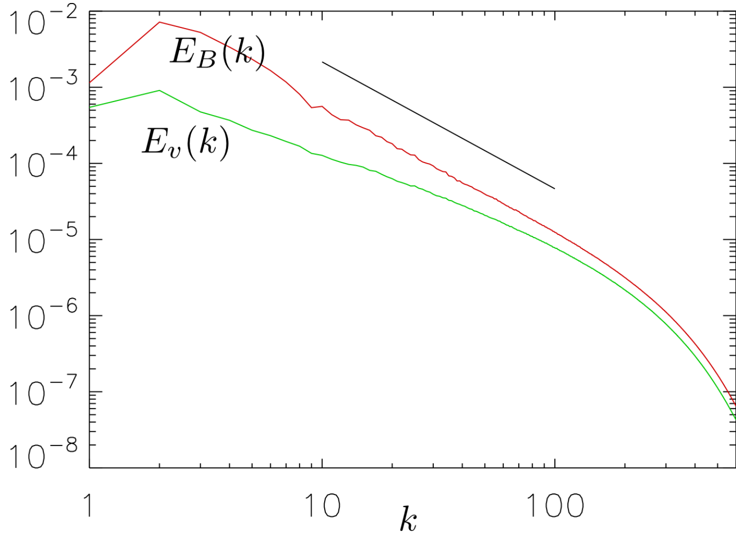




$v_r \approx 0.015 v_A$
$\epsilon \approx 0.006 \rho v_A^3$
$\frac{\Delta}{2}$
t
$\frac{v_r}{v_A}$
$S,\ S_4^{4/10} \times 10^4$
$768^3,\ S = 2.5 \cdot 10^4$
$1152^3,\ S = 4.4 \cdot 10^4$
$1536^3,\ S = 6.4 \cdot 10^4$
$1152^3,\ S_4^{4/10} = 3.6 \cdot 10^4$
$1536^3,\ S_4^{4/10} = 5.4 \cdot 10^4$

MHD turbulence in the current layer





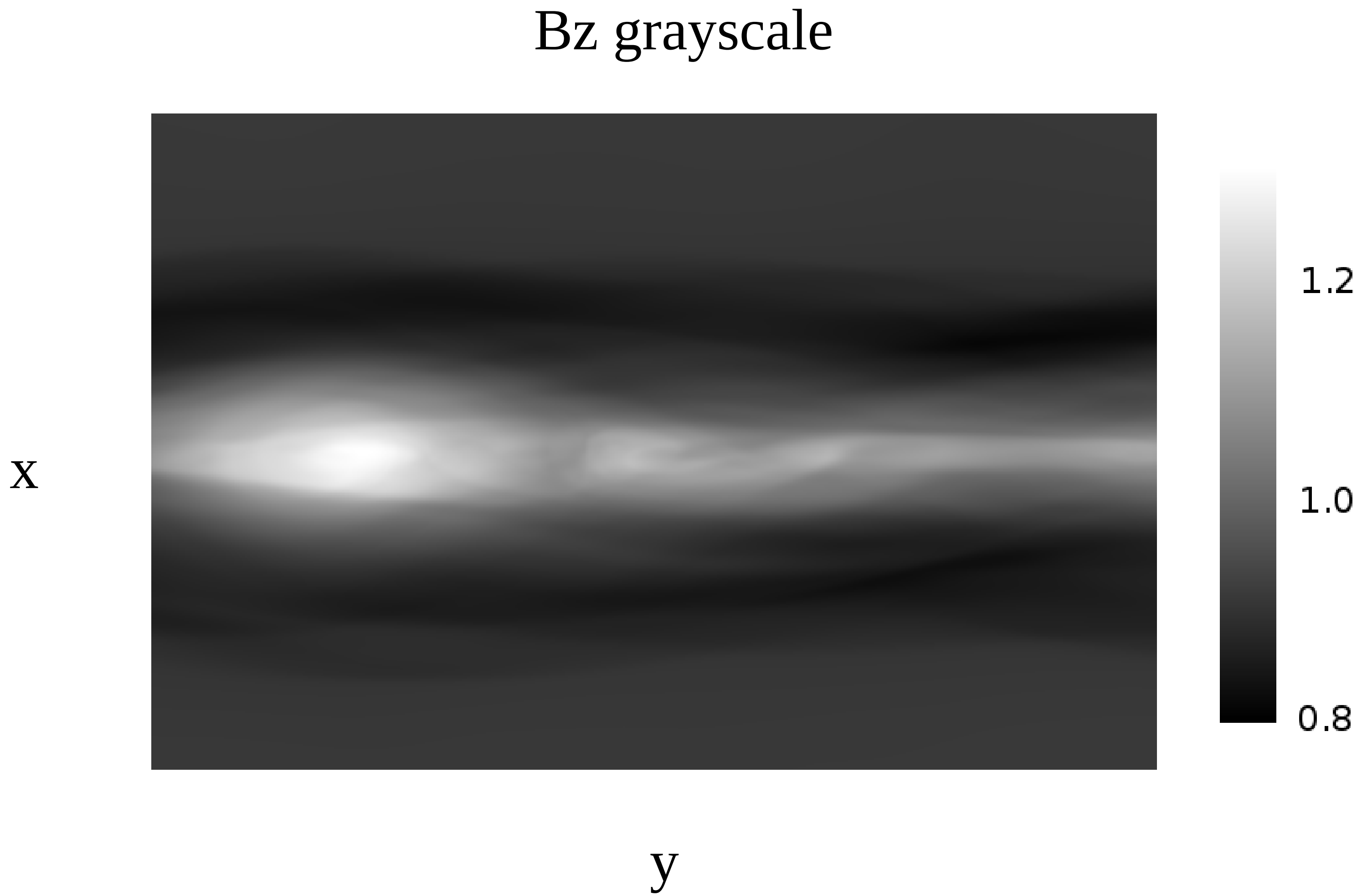
Anisotropic, in terms of spectra similar to decaying MHD turbulence, Eb>Ek, spectral slope: -1.7÷ -1.5

Energy content, out of 100% B_y free energy

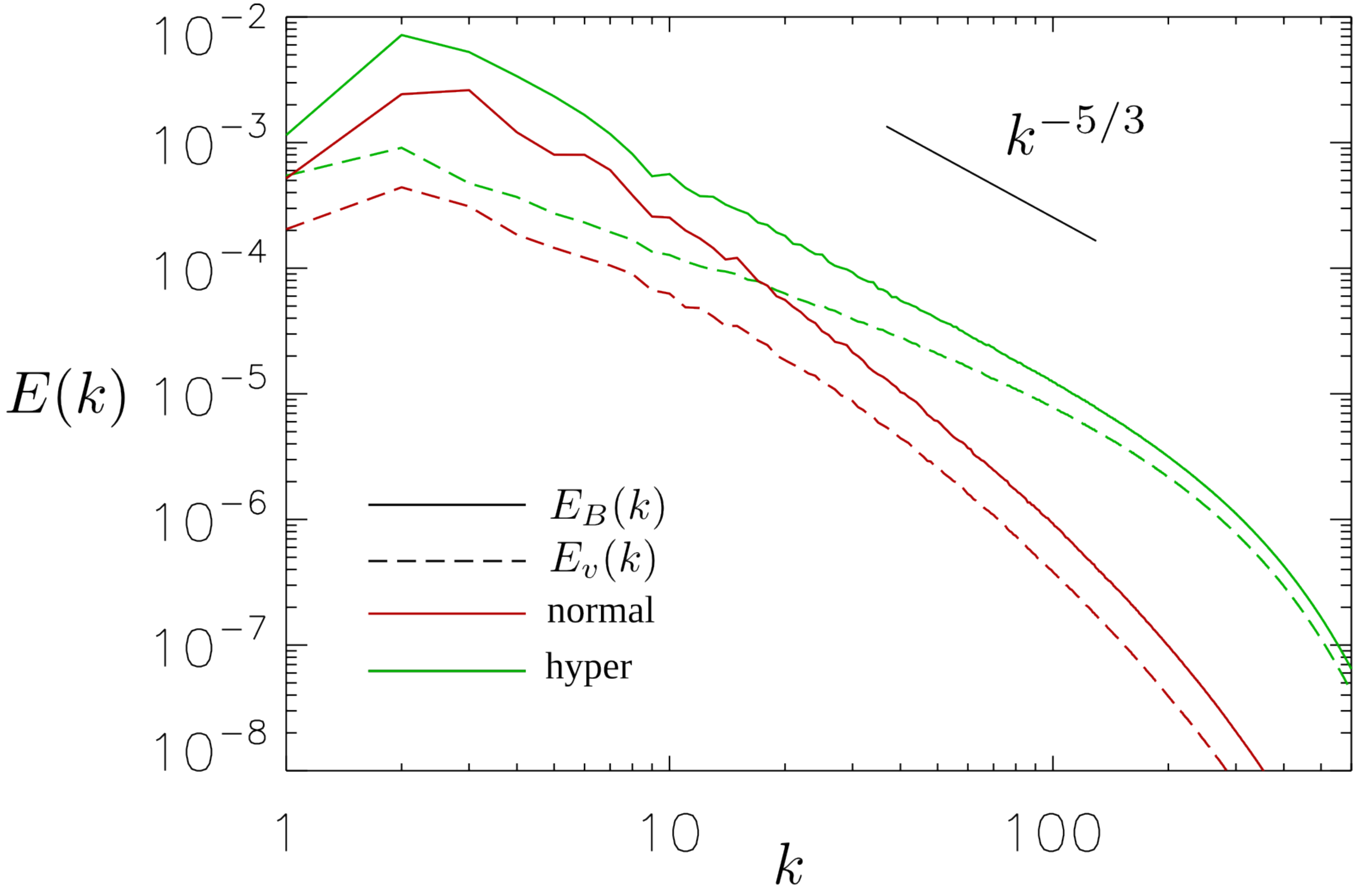
- 40% lost to dissipation
- 2% kinetic
- 2-4% δB_z
- 55% B_y and B_x (Alfven mode!)

Reconnection rate does not depend on B_z! (remember $v_A/\Lambda_{||}$ symmetry)

Turbulence is diamagnetic to Bz



Spectra indicate local-in-scale turbulence



This is why reconnection and dissipation rates flatten out in high-S limit

How can we explain X-ray flares?

Dissipation rate

$$\epsilon \leq 0.006 \rho v_A^3 / l_{SP}$$

Dissipation scale

$$\eta = (\nu^3/\epsilon)^{1/4}$$

is pushed below viscous scales and results in acceleration

This is why the ***magnitude*** of the dissipation rate is crucial.

Summary

• High resolution numerics are consistent with -5/3 slope and are inconsistent with -3/2 slope

• Anisotropy is consistent with *Godreich-Sridhar 1995* and is inconsistent with *Boldyrev 2006*

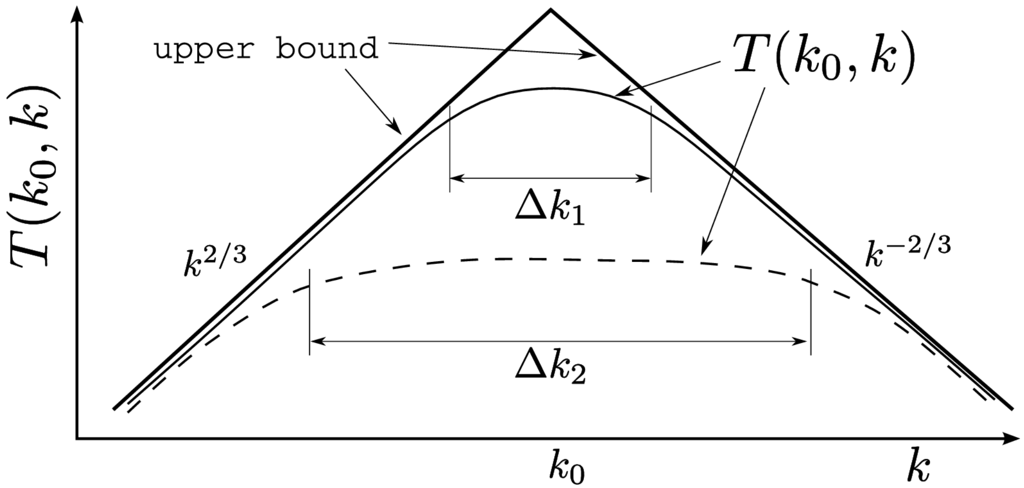
• The observed anisotropy in the imbalanced case is consistent with *AB & Lazarian (2008)*

• There is a lower limit to the reconnection rate and the heating rate per unit area in nearly-ideal MHD current layers.

• The minimum equilibrium current layer thickness, 0.015L, suggests that in most cases reconnection rate will be unaffected by plasma scales due to the scale-locality of turbulence.

• Electron acceleration is the result of pushing dissipation scale down to plasma scales

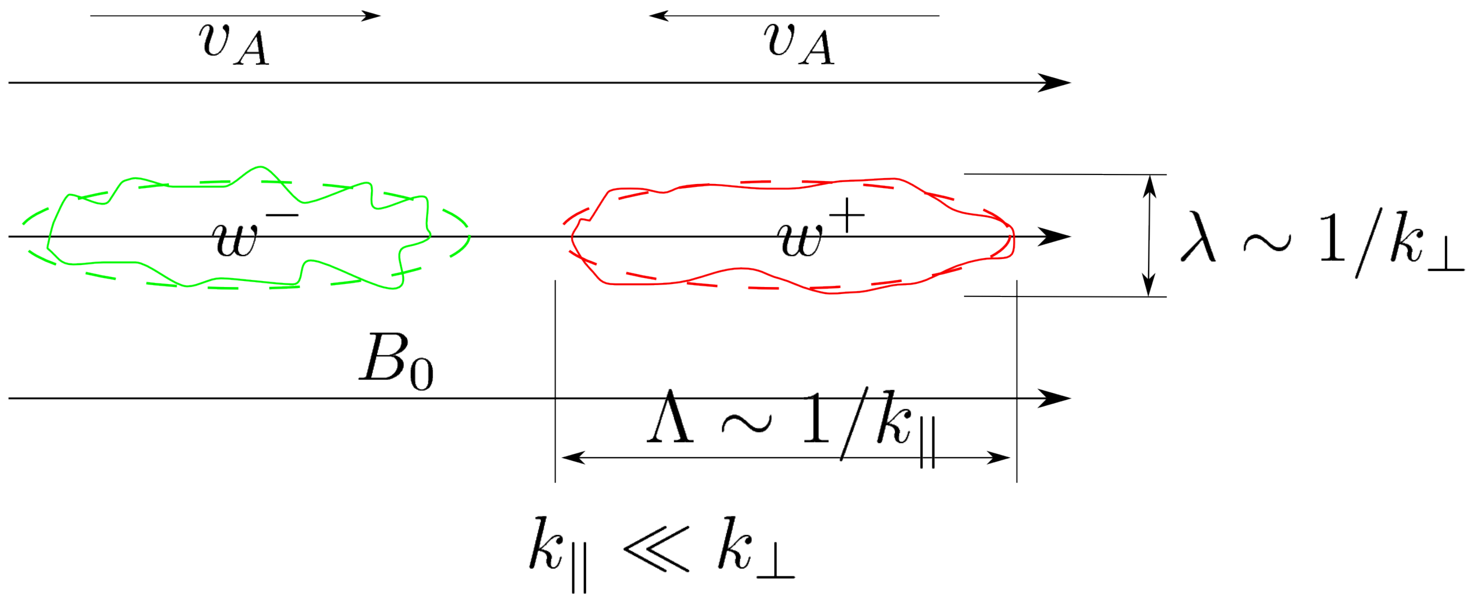
Locality of energy transfer



Diffuse locality of MHD turbulence is consistent with high value of the Kolmogorov constant. This explains wide transition towards asymptotic regime.

1) statistics of the asymptotic regime are very different from random,
2) it takes one order of magnitude in scale for turbulence to adjust
3) wider locality(x4.7 wider) explains lack of bottleneck in earlier numerics
4) nature's way of driving is probably even worse, so strange stuff in the solar wind is very expected

Basic properties of MHD turbulence



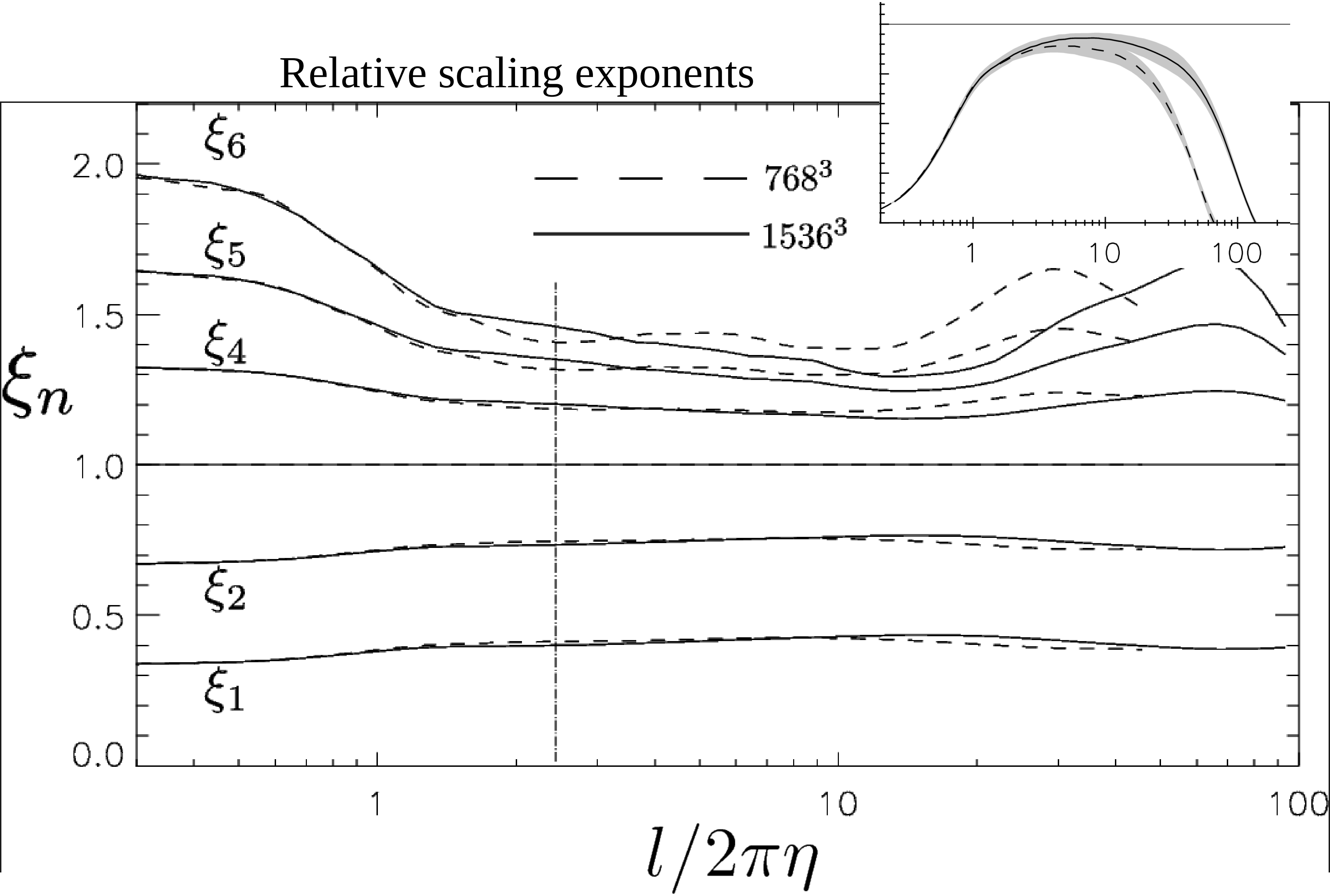
$$\partial_t \mathbf{w}_\perp^\pm \mp (\mathbf{v_A} \cdot \nabla_{||})\mathbf{w}_\perp^\pm + \hat{S}(\mathbf{w}_\perp^\mp \cdot \nabla_\perp)\mathbf{w}_\perp^\pm = 0$$

Contribution of nonlinear term has a tendency to increase, thus leading to "strong turbulence", despite a strong mean field, i.e. v_A>>w.

$$v_A k_{||}/\delta w k_\perp \sim 1$$

What do we know about intermittency in MHD case?

Relative scaling exponents

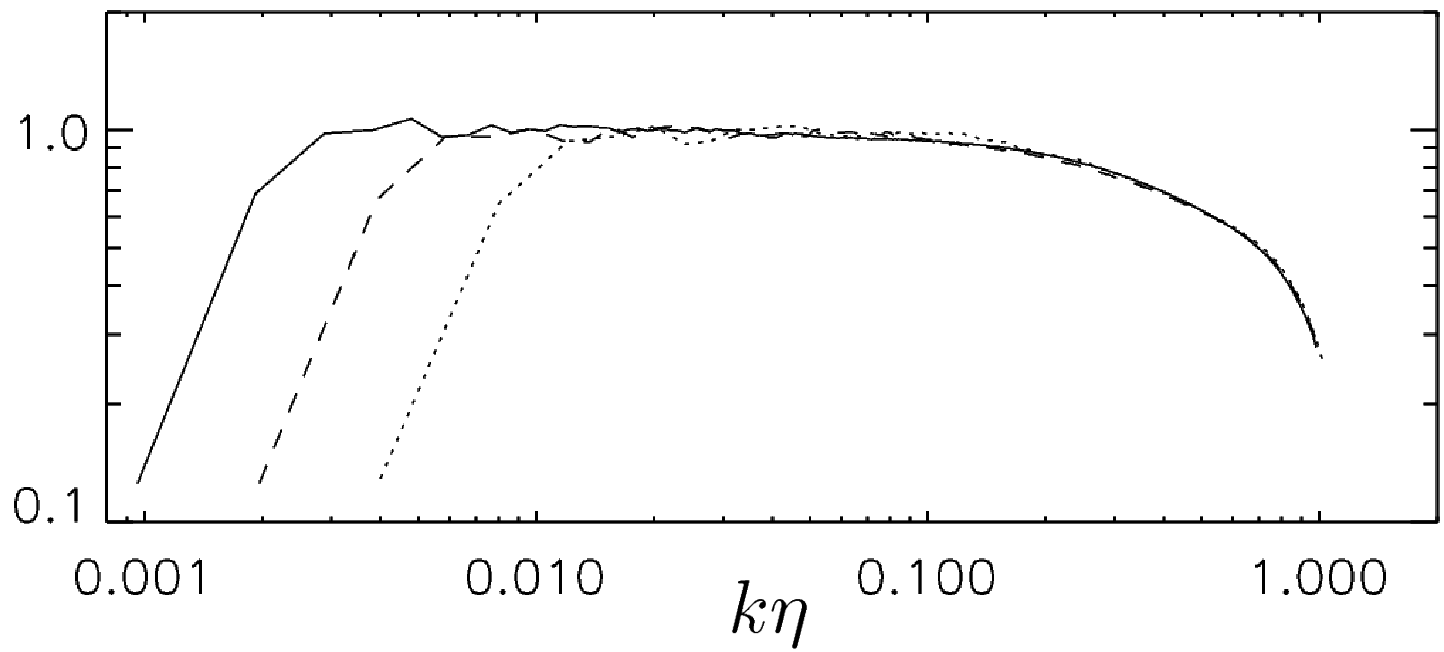


Residual Energy

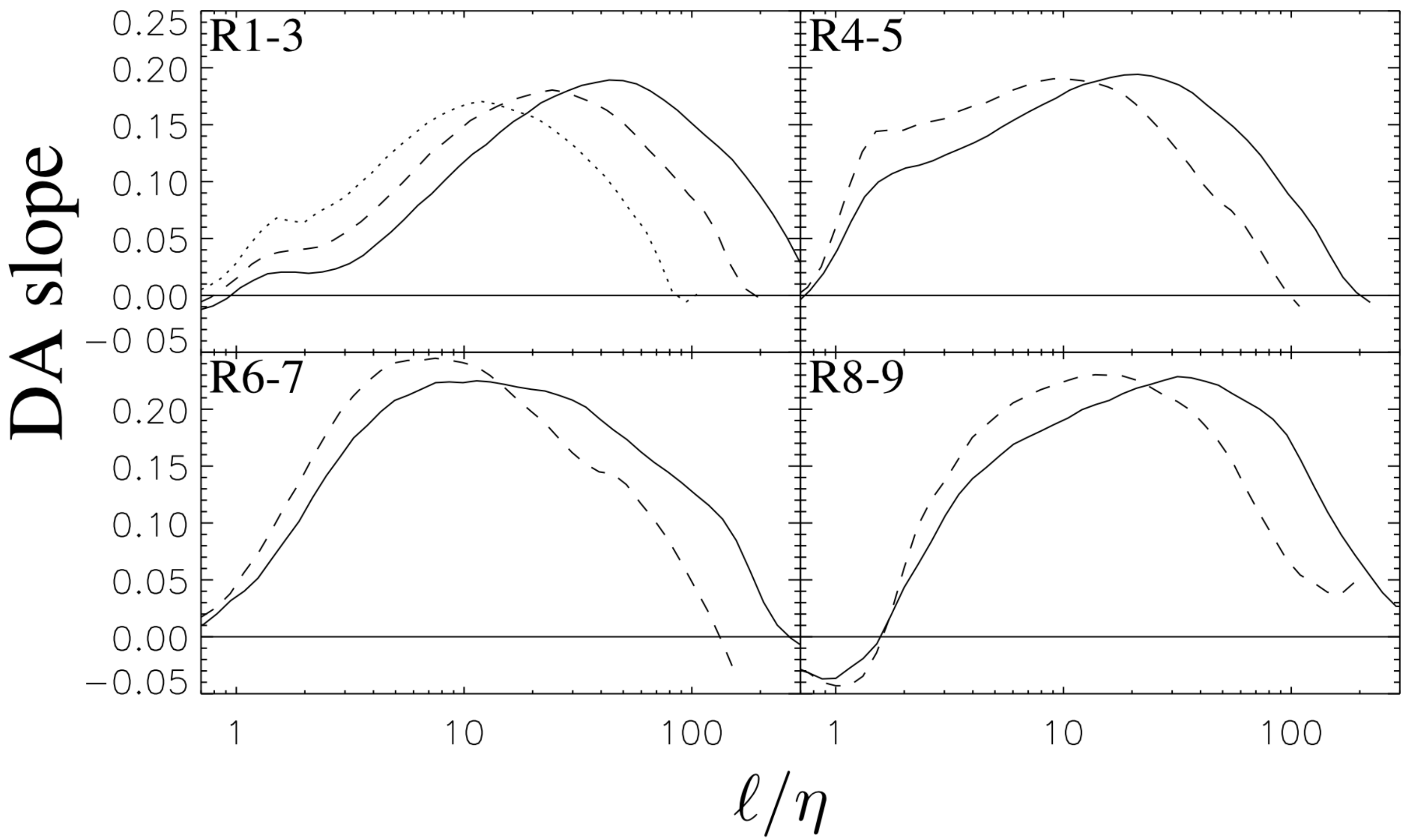
$$RE(k) = E_B(k) - E_v(k) \sim k^{-1.80}$$

previously suggested to have k^{-2} scaling by Mueller & Grappin (2005)

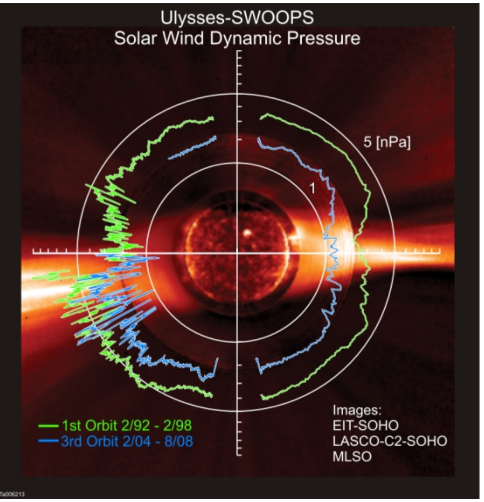
$$RE(k)k^{1.80}L^{0.13}\epsilon^{-2/3}$$



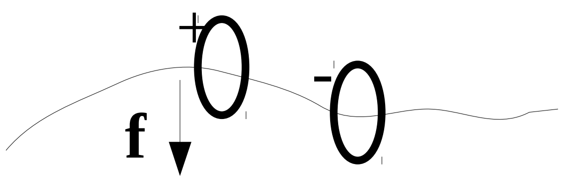
Alignment effects are pinned to the outer scale

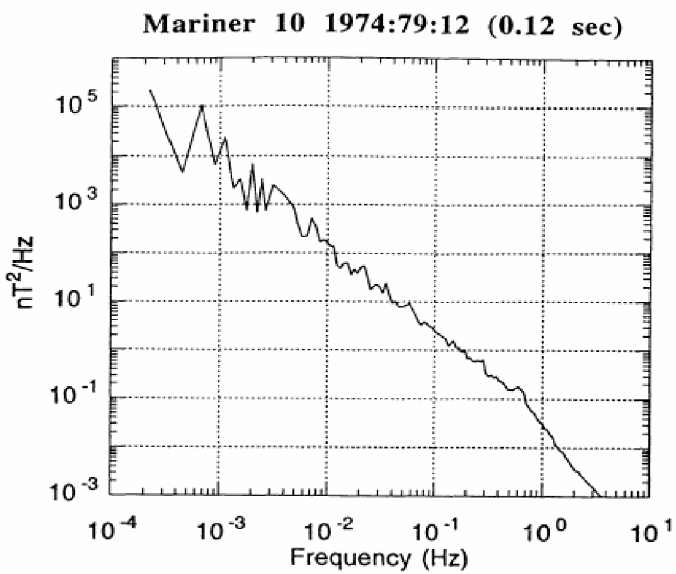


"Alfvenic" or Reduced MHD



- Could be derived for weakly collisional plasmas - Alfven mode does not require pressure support.
- Density fluctuations in the solar wind are much smaller than you would expect from transonic flow - it is mostly Alfvenic.





$$\partial_t \mathbf{w}_\perp^\pm \mp (\mathbf{v_A} \cdot \nabla_\parallel)\mathbf{w}_\perp^\pm + \hat{S}(\mathbf{w}_\perp^\mp \cdot \nabla_\perp)\mathbf{w}_\perp^\pm = 0$$

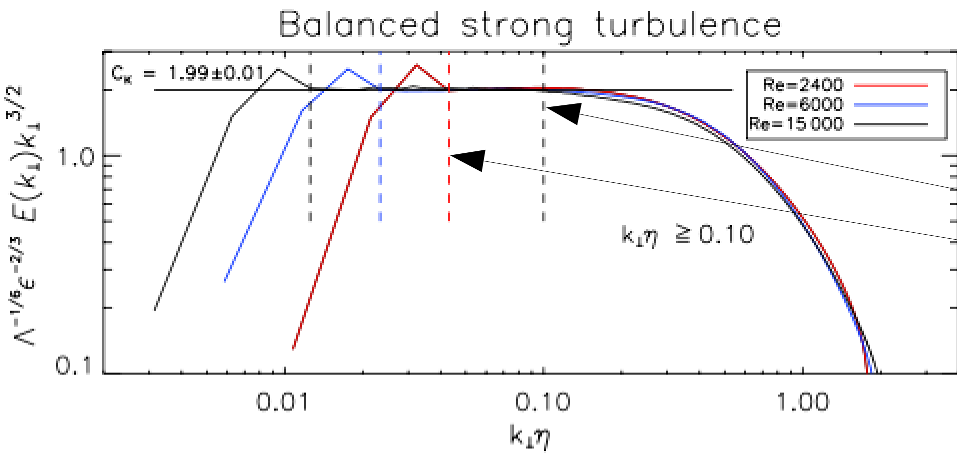
$$w \to wA, \quad t \to tB/A, \quad l_\perp \to l_\perp B, \quad l_\parallel \to l_\parallel B/A$$

$$(\text{also} \quad v_A \to v_A C, \quad \Lambda \to \Lambda C)$$

What about Perez et al, PRX 2012?

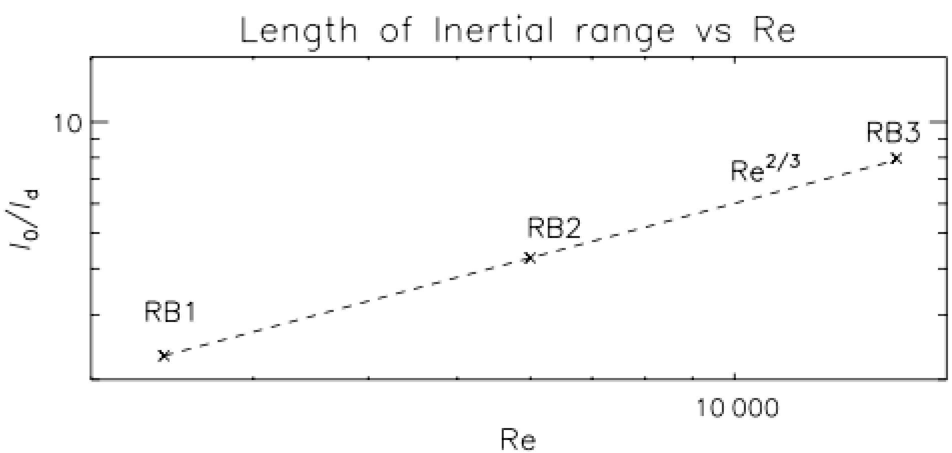
which claims that numerics is consistent with -3/2 model

Evidently, their analysis of numerics is deeply flawed:



Constant k is taken as the beginning of the inertial range and constant kη as the end of it. So the "length of the inertial range" is proportional to η. But η is just calculated by a formula:

$$\eta = \epsilon^{-2/9}\Lambda^{1/9}\nu^{2/3}$$



No wonder this "measurement" is consistent with the formula: it's the same formula that was used to calculate "data points"!

My comment to PRX is obviously due, will appear on astro-ph in a week or so.