Magnetically Controlled Flows for Hot Jupiter Winds and T Tauri Disk Accretion

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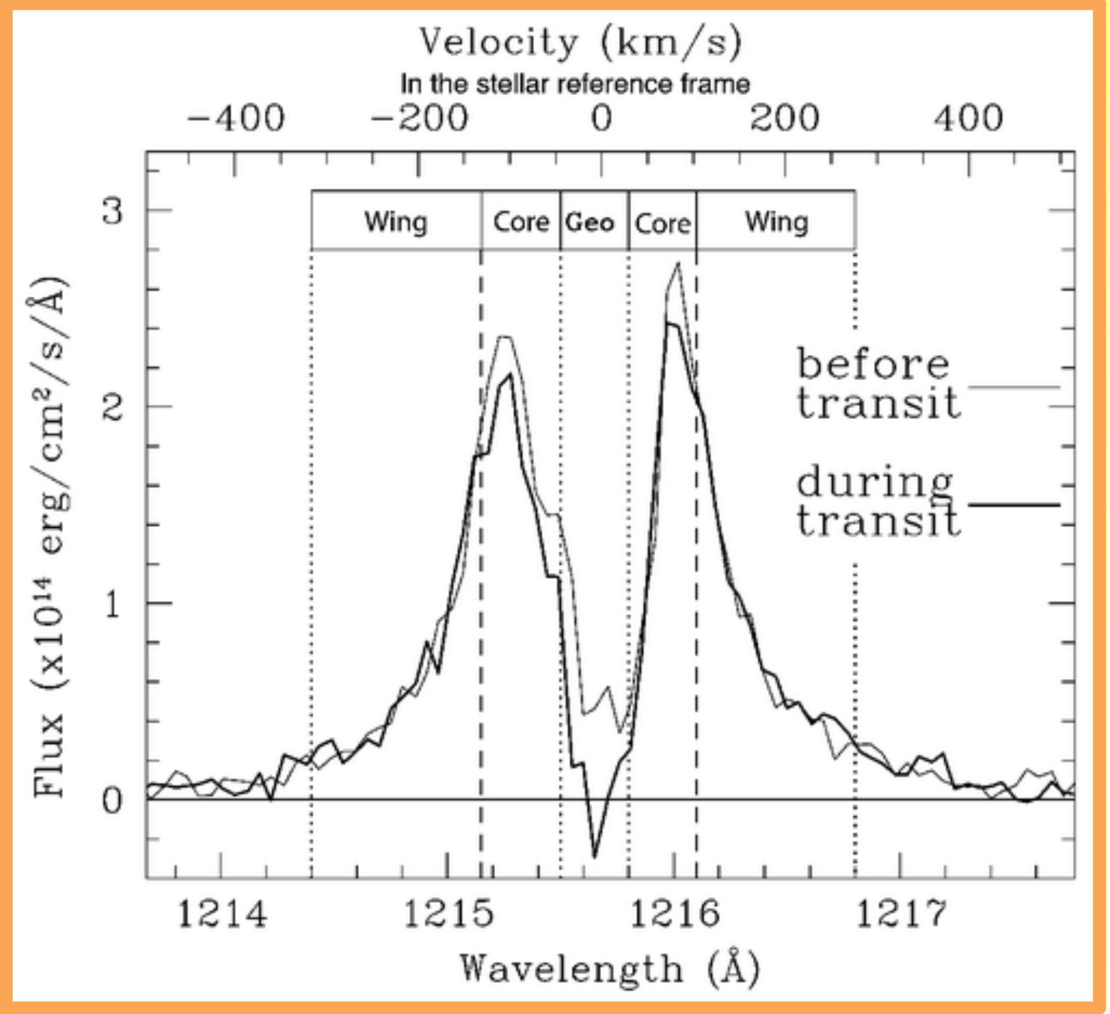
Magnetic Fields in the Universe IV
Playa Del Carmen, February 2013

Observations

- ***Two Hot Jupiter systems are observed(inferred) to have significant outflows from the planetary surface***
- ***T Tauri star/disks systems observed to have Octupole and Dipole Magnetic Fields, and support Transonic Flow from the Disk and onto Star***

THESE PHENOMENA ARE CONTROLLED BY MAGNETIC FIELDS

Magnetically Controlled Outflows from Hot Jupiters



Vidal-Madjar et al. 2008 ApJ

Hot Jupiters can Evaporate

- HD209458b (Vidal-Madjar et al. 2003, 2004; Desert et al. 2008; Sing et al. 2008; Lecavelier des Etangs et al. 2008)
- HD189733b (Lecavelier des Etangs et al. 2010)

$$\frac{dM}{dt} = 10^{10} - 10^{11} g/s$$

Planetary System Parameters

$$M_* = 1 M_{SUN} \quad F_{UV} \approx 100 - 1000 \ (cgs)$$

$$M_P \approx 1 M_{JUP} \quad R_P \approx 1.4 R_{JUP}$$

$$B_* \approx 1 \ Gauss \quad B_P \approx 1 \ Gauss$$

$$\varpi_{orb} \approx 0.05 AU \quad P_{orb} \approx 4 day \quad e = 0$$

$$\varpi_{orb} \approx 10 R_* \approx 100 R_P \quad \varpi_{orb} >> R_* >> R_P$$

Basic Regime of Operation

$$\frac{dM}{dt} = \eta \frac{\pi R_P^3 F_{UV}}{G M_P} \approx 10^{10}\, g\ s^{-1} \approx 10^{-4}\ \ M_J\ \ Gyr^{-1}$$

$$\frac{B^2}{8\pi\rho v^2} \approx 10^4 - 10^6\ \ (magnetically - controlled)$$

$$\frac{\omega_C}{\Gamma} = \frac{qB}{cmn\sigma v} \approx 10^4\ \ (well - coupled)$$

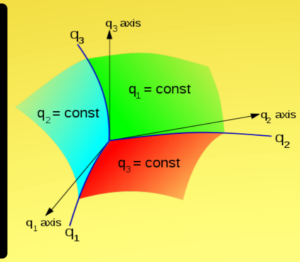
$$\frac{B_\perp}{B} = \mathrm{O}\left(8\pi\rho v^2 / B^2\right) < 10^{-4}\ \ (current - free)$$

TWO COUPLED PROBLEMS

- LAUNCH of the outflow from planet
- PROPAGATION of the outflow in the joint environment of star and planet, including gravity, stellar wind, stellar magnetic field
- Matched asymptotics: Outer limit of the inner problem (launch of wind) provides the inner boundary condition for the outer problem (propagation of wind)

This Work Focuses on Launch of the Wind

Construct Coordinate Systems that follow Magnetic Field Lines



Basis Vectors

covariant:

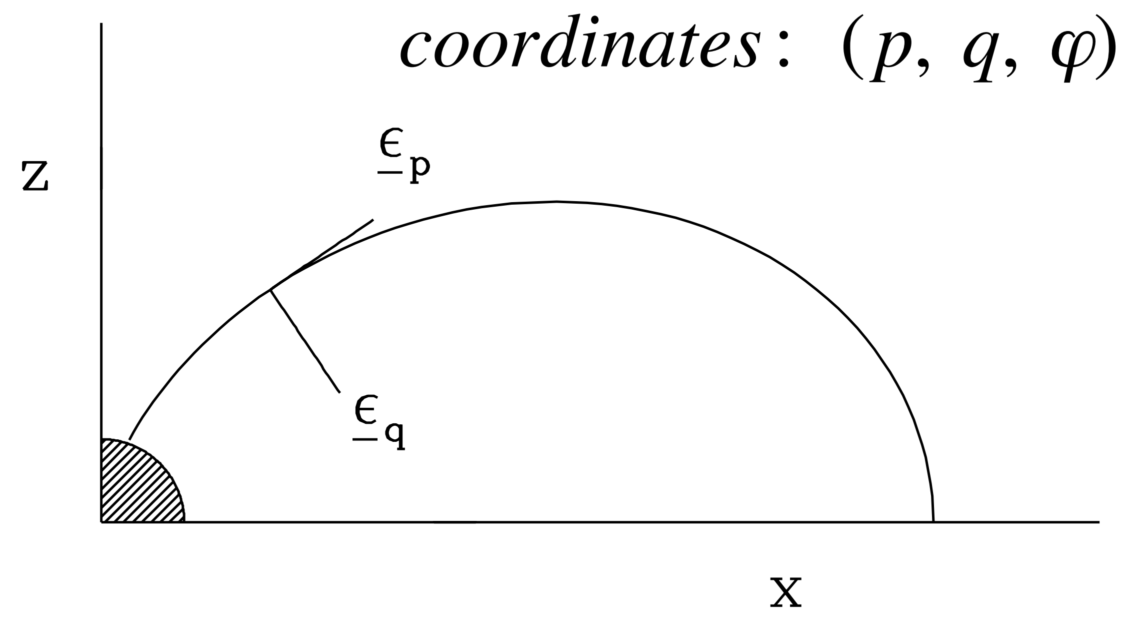
$$\underline{\varepsilon}_p = \nabla p$$

$$\underline{\varepsilon}_q = \nabla q$$

contravariant:

$$\vec{e}_p = \partial \vec{r} / \partial p$$

$$\vec{e}_q = \partial \vec{r} / \partial q$$



Scale Factors: $h_j = \left|\underline{\varepsilon}_j\right|^{-1}$

Unit Vectors: $\hat{e}_j = \underline{\varepsilon}_p \, h_j = \vec{e}_j \, h_j^{-1}$

The Coordinate System

$$\vec{B} = B_P\left[\xi^{-3}\left(3\cos\theta\ \hat{r} - \hat{z}\right)\right] + B_*(R_*/\varpi)^3\,\hat{z}$$

$$p = \left(\beta\xi - \xi^{-2}\right)\cos\theta \qquad q = \left(\beta\xi^2 + 2/\xi\right)^{1/2}\sin\theta$$

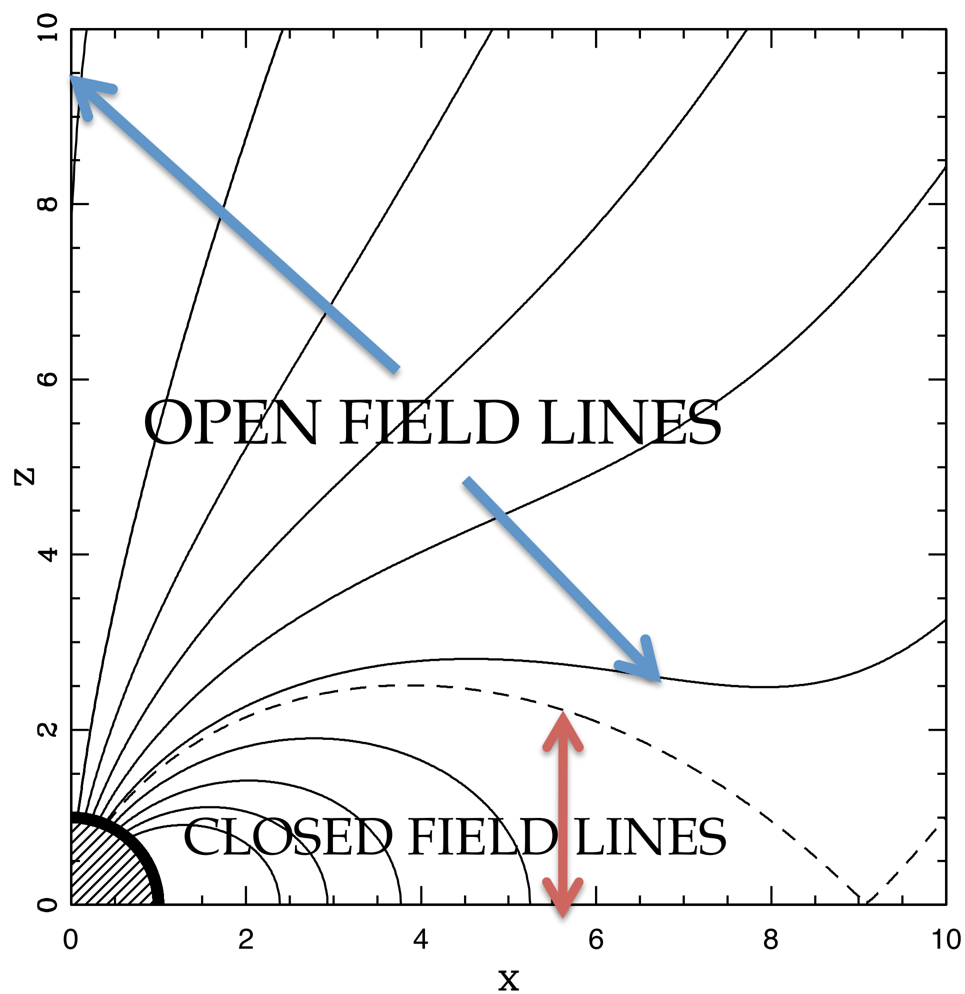
$$where \quad \beta = (B_* R_*^3/\varpi^3)/B_P \approx 10^{-3} \quad and \quad \xi = r/R_*$$

$$\nabla p = f(\xi)\cos\theta\ \hat{r} - g(\xi)\sin\theta\ \hat{\theta}$$

$$\nabla q = \left[g(\xi)\sin\theta\ \hat{r} + f(\xi)\cos\theta\ \hat{\theta}\right] g^{-1/2}(\xi)$$

$$where \quad f = \beta + 2\xi^{-3} \quad and \quad g = \beta - \xi^{-3}$$

Magnetic Field Configuration



Magnetic field lines are lines of constant coordinate q.
The coordinate p measures distance along field lines.
Field lines are open near planetary pole and are closed near the equator. Fraction of open field lines:

$$f = 1 - \left[1 - \frac{3\beta^{1/3}}{2+\beta} \right]^{1/2}$$

Equations of Motion

$$u = \frac{v}{a_S} \qquad \alpha = \frac{\rho}{\rho_1} \qquad \psi = \frac{\Psi}{a_S^2} \rightarrow \frac{b}{\xi} \qquad b \equiv \frac{GM_P}{R_P a_S^2}$$

Steady-state flow along field-line direction:
Fluid fields are functions of coordinate p only.

$$\alpha \frac{\partial u}{\partial p} + u \frac{\partial \alpha}{\partial p} = -\frac{\alpha u}{h_q h_\phi} \frac{\partial}{\partial p}\left(h_q h_\phi\right)$$

$$u \frac{\partial u}{\partial p} + \frac{1}{\alpha} \frac{\partial \alpha}{\partial p} = -\frac{\partial \psi}{\partial p} = -\frac{\partial \psi}{\partial \xi} \frac{\partial \xi}{\partial p}$$

Two parameters specify the dimensionless problem

$$b \equiv \frac{GM_P}{R_P a_S^2} \approx 10$$

$$\beta \equiv \frac{B_*\left(R_* / \varpi\right)^3}{B_P} \approx 0.001$$

Solutions

$$\frac{b}{3} = \frac{2f^2 - \left(g^2/f + 2g + 2f\right)q^2/\xi^2}{f^3\xi^2 + \left(g^2 - f^2\right)q^2}$$

Sonic point

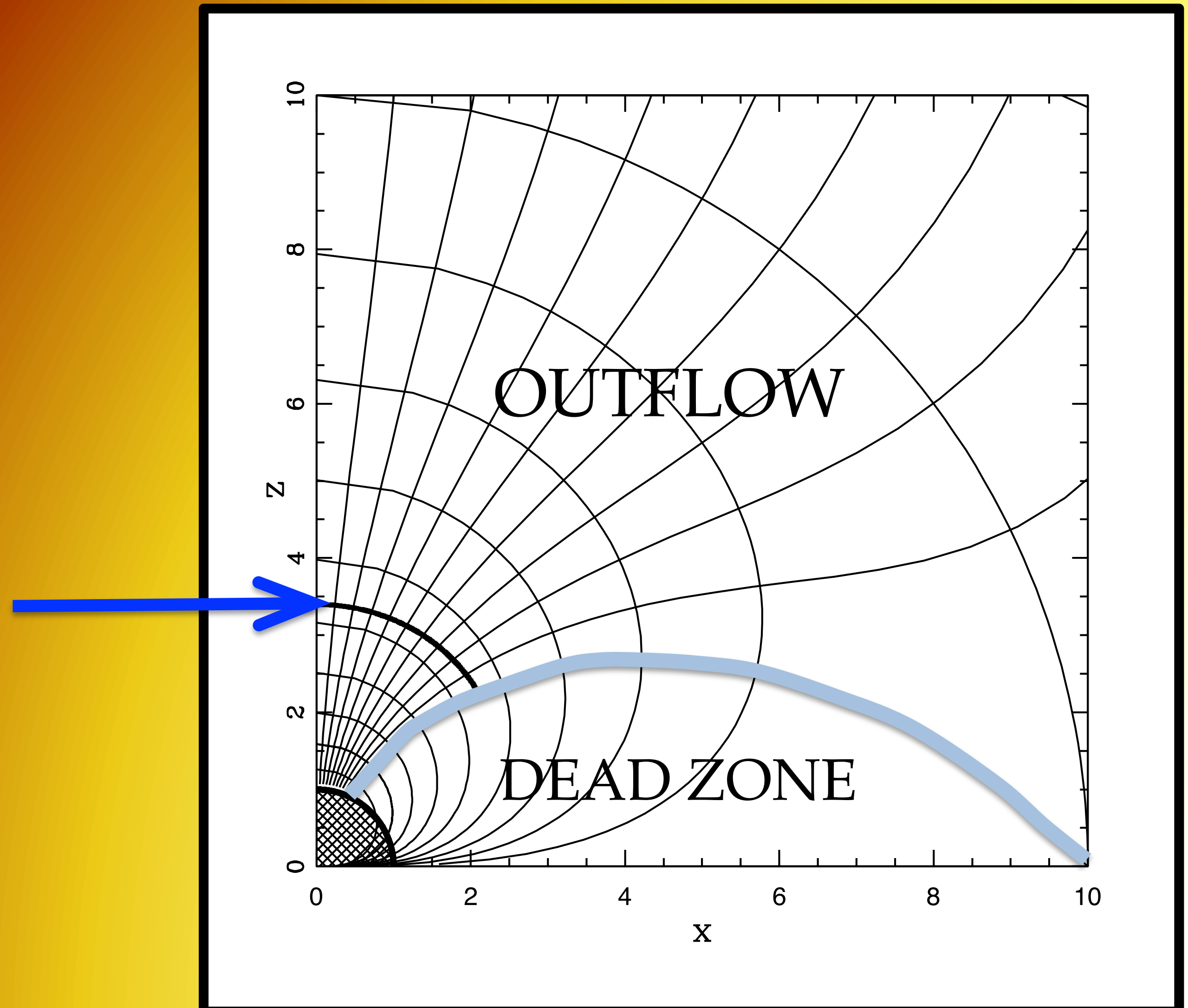
$$\lambda = qH_S^{-1/2}\exp\left[\frac{\lambda^2 H_1}{2q^2} + \frac{b}{\xi_S} - b - \frac{1}{2}\right]$$

Continuity eq. constant

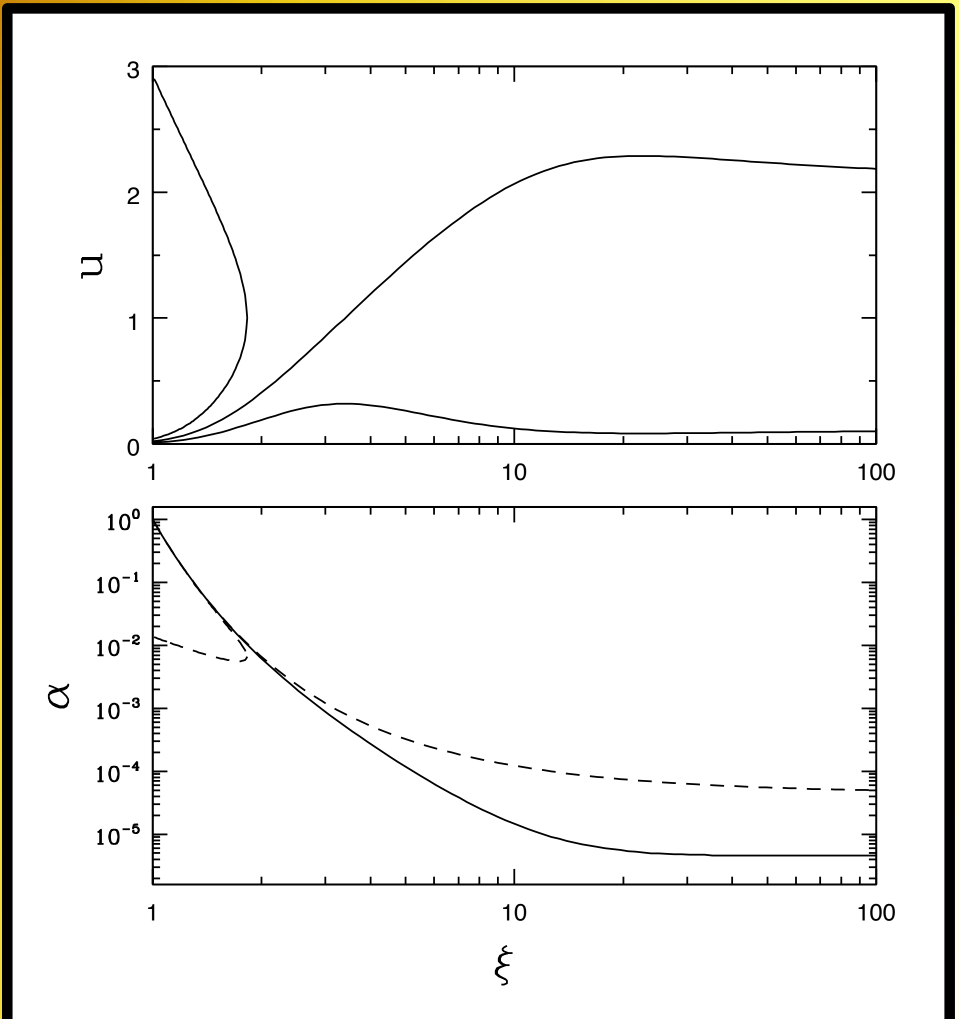
$$f = \beta + 2\xi^{-2},\ \ g = \beta - \xi^{-3},\ \ and$$

$$H = f^2\cos^2\theta + g^2\sin^2\theta,\ \ \sin^2\theta = q^2/(\beta\xi^2 + 2/\xi)$$

Sonic Surface

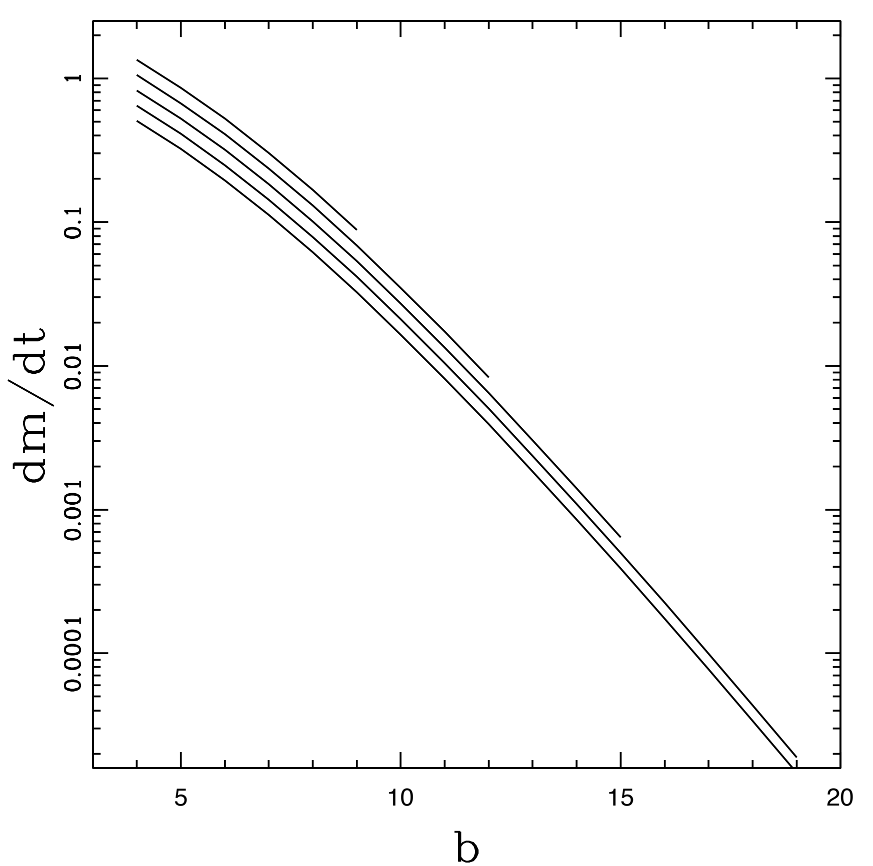


Fluid Field Solutions



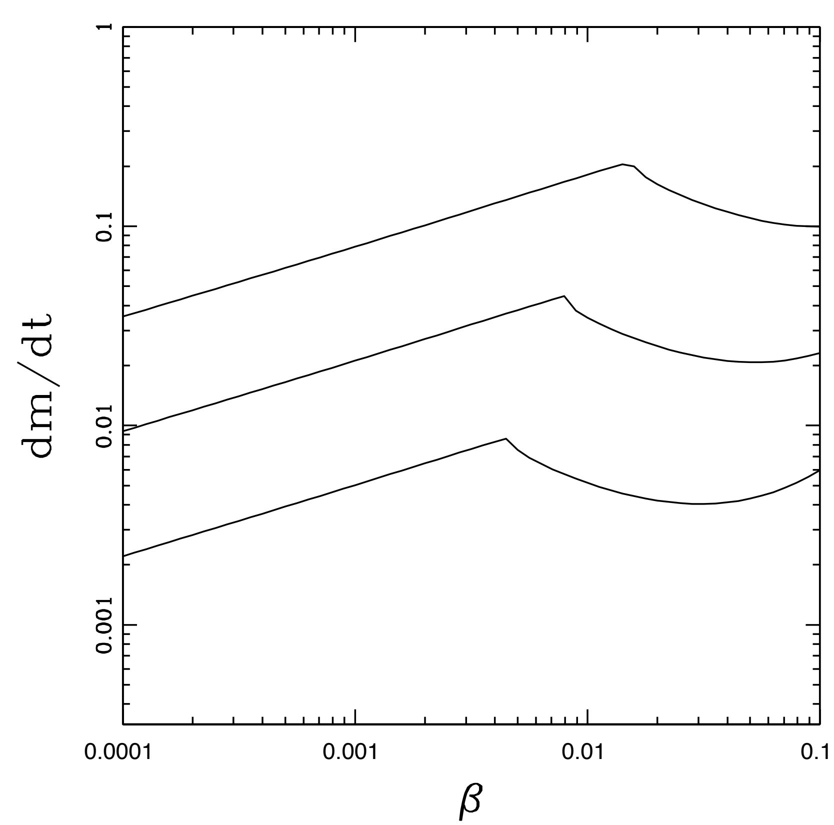
Mass Outflow Rates

$$\frac{dm}{dt} = 4\pi \int_0^{q_X} \lambda \, dq$$

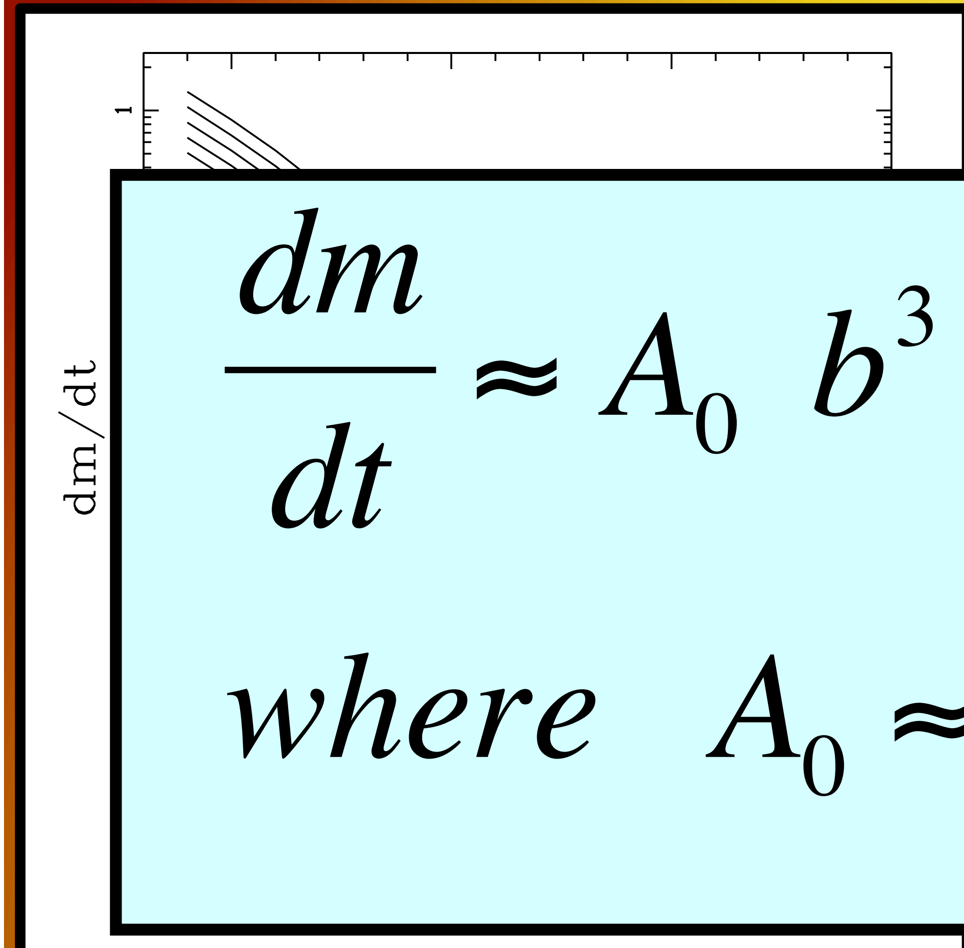


$$b = GM_P / a_S^2 R_P$$

$$\beta = (B_* R_*^3 / \varpi^3) / B_P$$



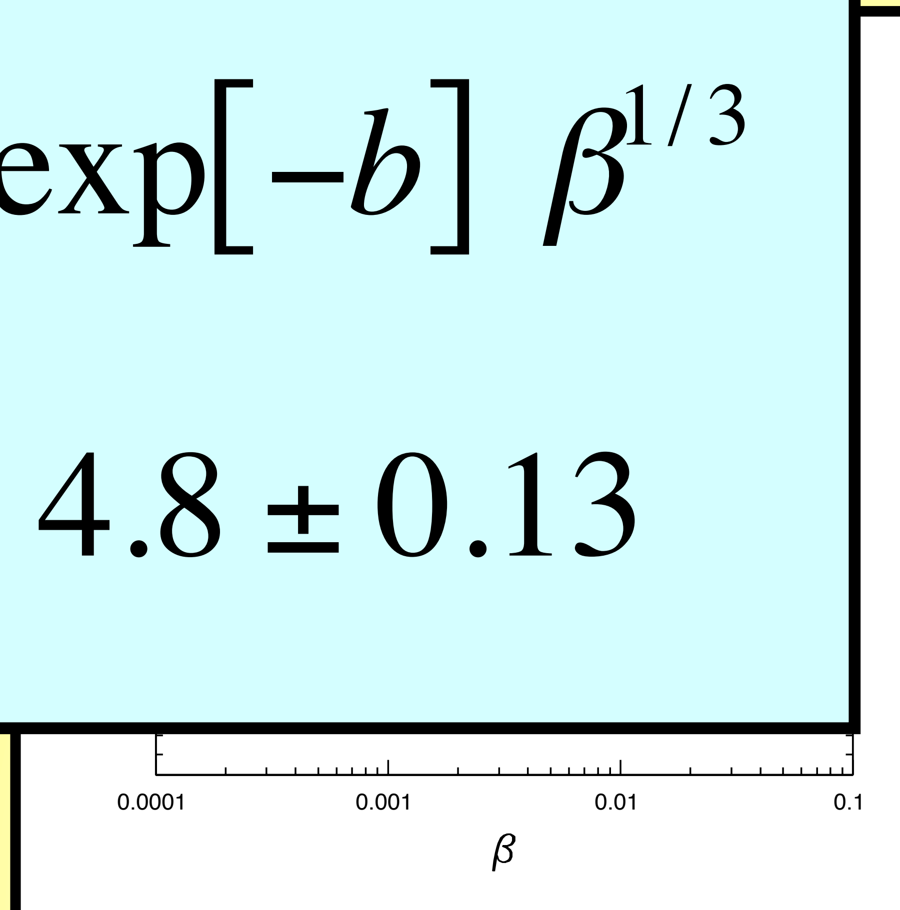
Mass Outflow Rates



$$\beta = (B_* R_*^3 / \varpi^3) / B_P$$

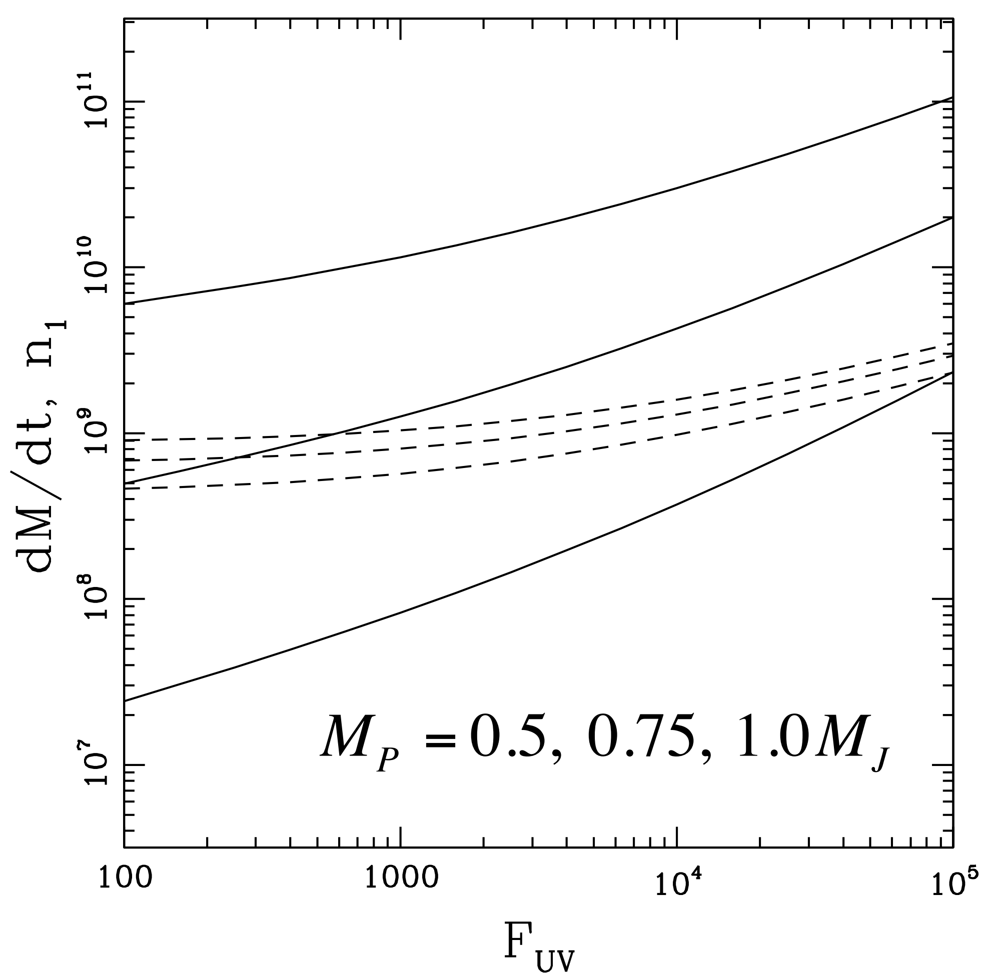
$$\frac{dm}{dt} \approx A_0 \; b^3 \exp[-b] \; \beta^{1/3}$$

$$where \quad A_0 \approx 4.8 \pm 0.13$$

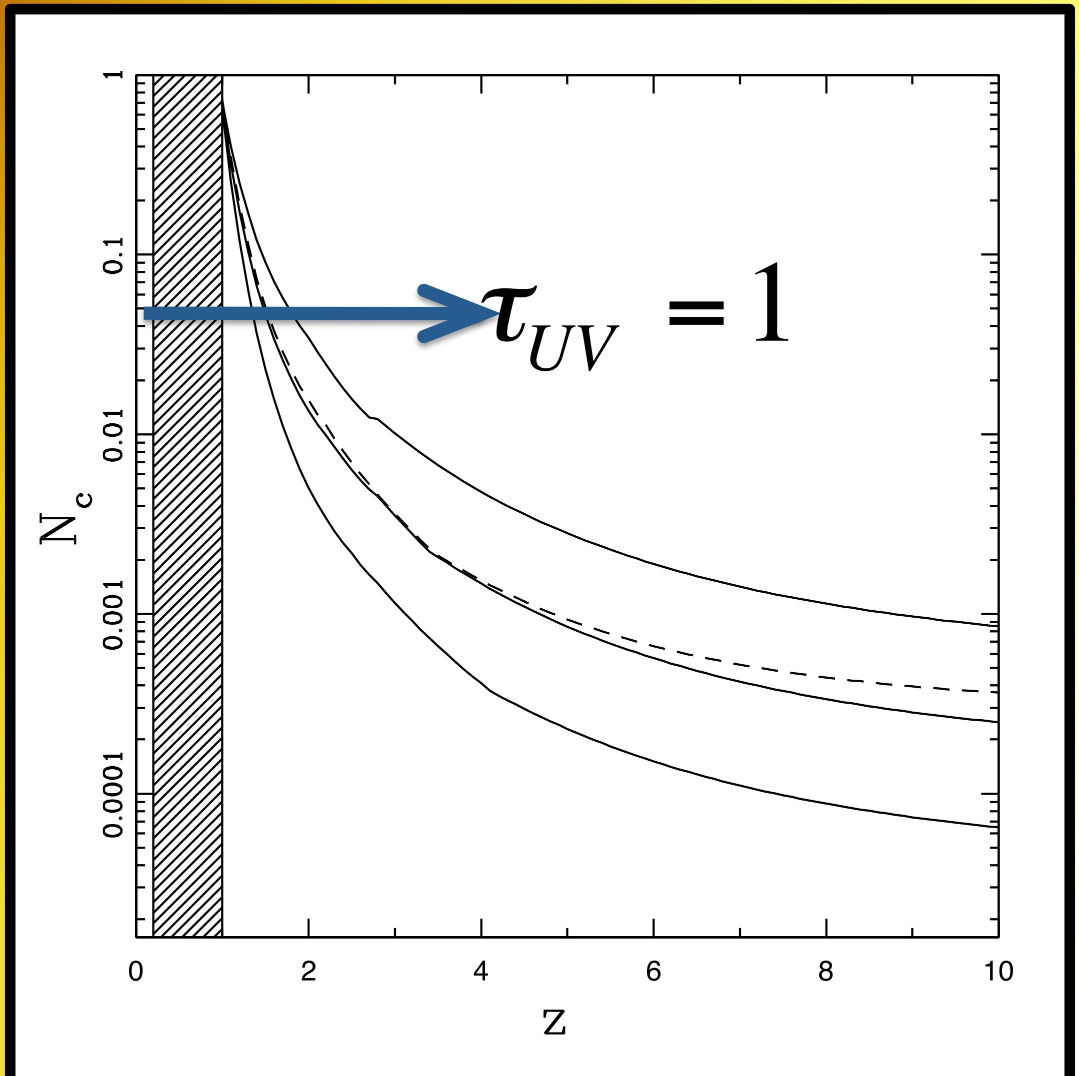


$$b = GM_P / a_S^2 R_P$$

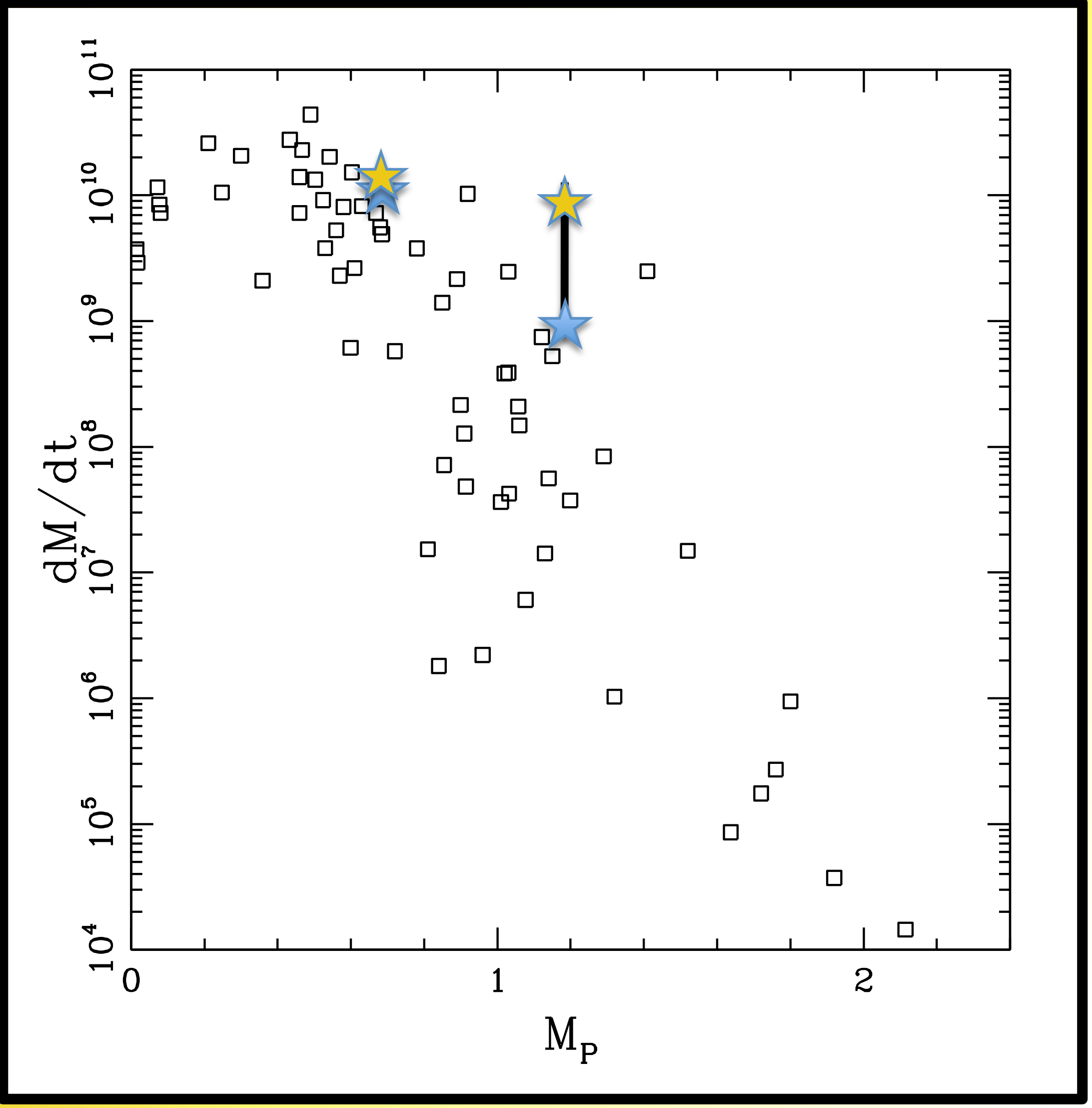
Physical Outflow Rate vs Flux



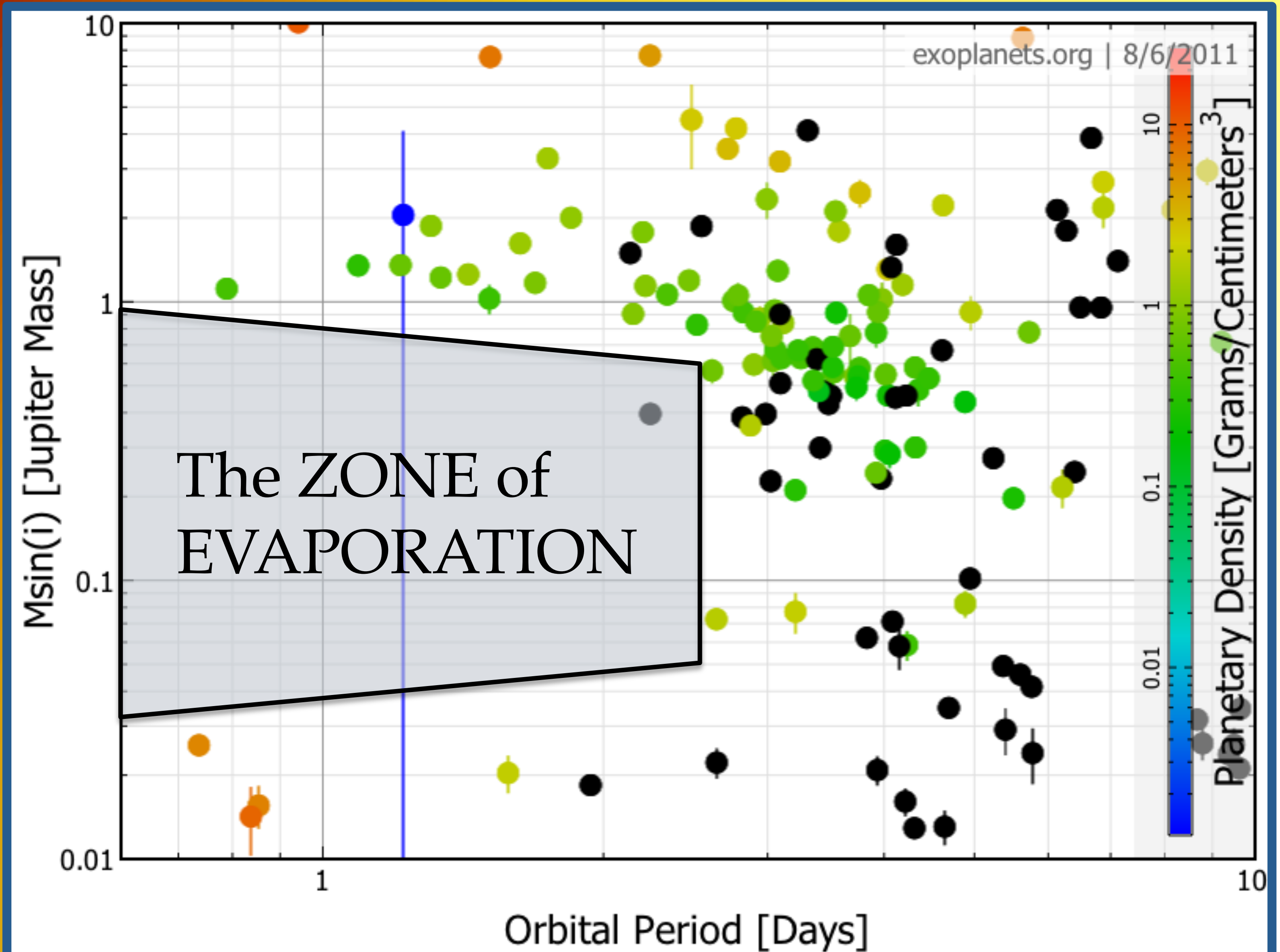
Column Density



Observational Implications

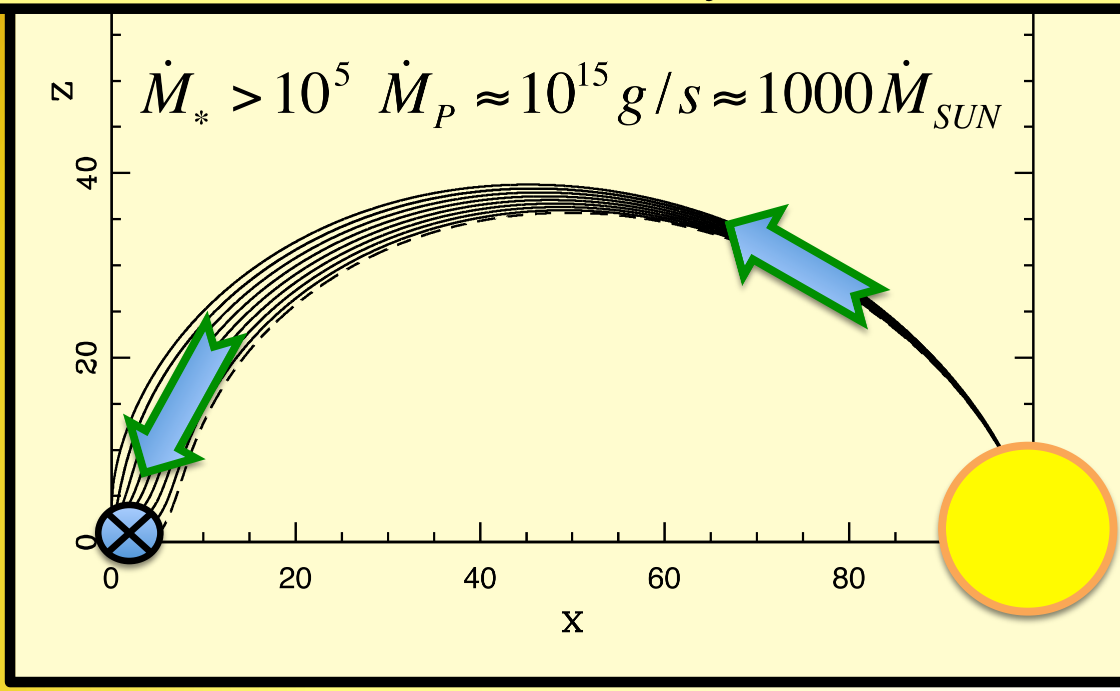


Implication/Prediction



The Extreme Regime

In systems where the stellar outflow and the stellar magnetic field are both sufficiently strong, the Planet can Gain Mass from the Star



Summary 1.0

- Planetary outflows magnetically controlled
- Outflow rates are moderately *lower*
- Outflow geometry markedly different:
  Open field lines from polar regions
  Closed field lines from equatorial regions
- In extreme regime with strong stellar outflow the planet could gain mass from the star
- Outflow rates sensitive to planetary mass:

$$\dot{m} \propto b^3 \exp\left[-b\right] \beta^{1/3}, \qquad b = GM_P/(a_S^2 R_P)$$

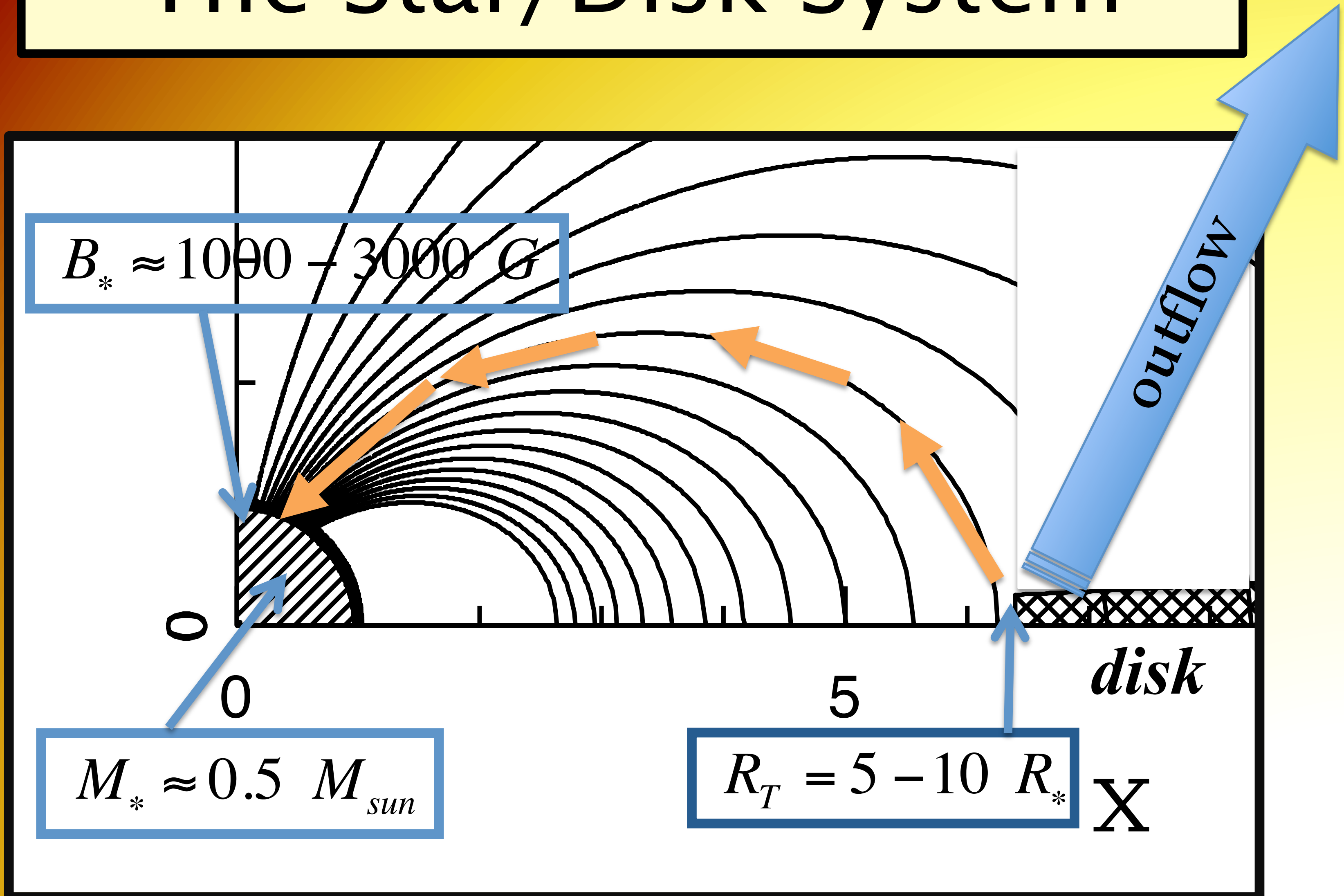
(F. C. Adams, 2011, ApJ, 730, 27)

Magnetically Controlled Accretion Flows onto TTStars

- Most material that ends up in a forming star is processed through the disk; final accretion flow occurs via magnetic field
- ***T Tauri star/disks systems observed to have Octupole and Dipole Magnetic Fields, and support Transonic Flow onto Star***
- Want to understand the sonic transition for magnetically controlled flows in general

(F. Adams & S. Gregory, 2012, ApJ, 744, 55)

The Star/Disk System



Basic Regime of Operation

$$\frac{dM}{dt} \approx 10^{-7} - 10^{-8} \;\; M_{sun} yr^{-1}$$

$$\frac{B^2}{8\pi\rho v^2} \approx 350 - 3000 \;\; (magnetically - controlled)$$

$$\frac{\omega_C}{\Gamma} = \frac{qB}{cmn\sigma v} \approx 10^4 - 10^5 \;\; (well - coupled)$$

$$\frac{B_\perp}{B} = \mathrm{O}\left(8\pi\rho v^2 / B^2\right) < 10^{-3} \;\; (current - free)$$

Equations of Motion

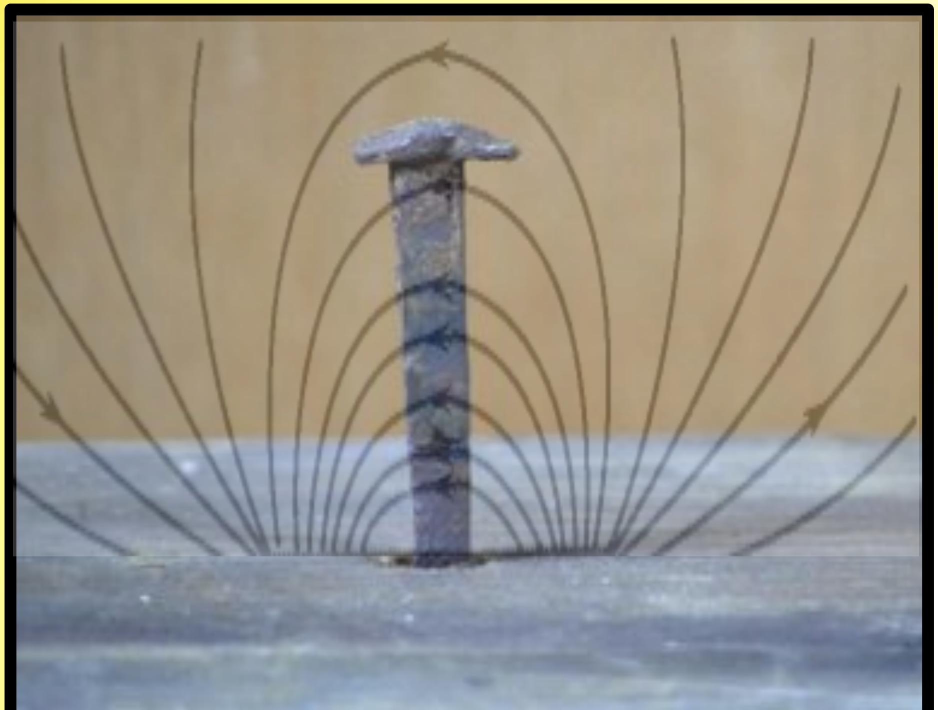
Steady-state flow, polytropic equation of state:

$$\nabla \bullet \left(\rho \, \vec{u}\right) = 0 \qquad\qquad P = K \, \rho^{1+1/n}$$

$$\vec{u} \bullet \nabla \vec{u} + \nabla \Psi + \frac{1}{\rho} \nabla P + \vec{\Omega} \times \left(\vec{\Omega} \times \vec{r}\right) = 0$$

$$\vec{B} = \kappa \, \rho \, \vec{u} \qquad where \qquad \kappa = const$$

TO A MAN WITH
A HAMMER,
EVERYTHING LOOKS
LIKE A NAIL



Dimensionless Equations of Motion

$$\xi \equiv \frac{r}{R_*}, \quad u \equiv \frac{|\vec{u}|}{a_S}, \quad \alpha \equiv \frac{\rho}{\rho_1}, \quad \psi \equiv \frac{\Psi}{a_S^2}, \quad b \equiv \frac{GM_P}{R_* a_S^2}, \quad \omega \equiv \left(\frac{\Omega R_*}{a_S}\right)^2$$

Steady-state flow along field-line direction, Fluid fields are functions of coordinate p only:

$$\alpha \frac{\partial u}{\partial p} + u \frac{\partial \alpha}{\partial p} = -\frac{\alpha u}{h_q h_\phi} \frac{\partial}{\partial p}\left(h_q h_\phi\right)$$

$$u \frac{\partial u}{\partial p} + \frac{\alpha^{1/n}}{\alpha} \frac{\partial \alpha}{\partial p} - \omega \xi \sin\theta \left|\nabla p\right|^{-1} \left(\hat{x} \cdot \hat{p}\right) = -\frac{\partial \psi}{\partial p}$$

Integrated Equations of Motion

$$\alpha \; u \; h_q h_\phi = \lambda \qquad \left(4\pi\rho v r^2 = dM/dt\right)$$

$$\frac{1}{2}u^2 + n\alpha^{1/n} + \psi - \omega \; I = \mathrm{E}$$

$$I = \int \; \xi \sin\theta \; \left|\nabla p\right|^{-1} \left(\hat{x}\cdot \; \hat{p}\right) \; dp$$

$$\lambda = mass \quad accretion \quad rate$$

$$\mathrm{E} = energy$$

Sonic Transition Condition

$$\alpha^{1/n}\,\frac{\Upsilon}{\xi} + \omega\ \Lambda \ = \ \frac{b}{\xi^2}$$

$$\Upsilon(\xi,\theta) \equiv \frac{\xi}{h_p}\,\frac{\partial h_p}{\partial \xi} \qquad (\mathit{where}\quad h_p = h_q h_\phi)$$

$$\Lambda(\xi,\theta) \equiv \xi \sin\theta \left(\frac{\partial p}{\partial \xi}\right)^{-1} \left(\hat{x}\cdot \nabla p\right)$$

Reduce to old/known result:

$$\alpha^{1/n}\,\frac{\Upsilon}{\xi} + \omega\,\Lambda = \frac{b}{\xi^2} \quad \rightarrow \quad \xi_s = \frac{b}{2}$$

Shu (1992) *Gas Dynamics*, *p*. 346:

$$n \rightarrow \infty,\ \omega \rightarrow 0,\ \Upsilon \rightarrow 2 \quad \Rightarrow \quad 2 = \frac{b}{\xi}$$

Score Card

b *and* ω: *system parameters* $(n \to \infty)$

λ *and* E: *conserved quantities*

$\Upsilon(\xi,\theta)$ *and* $\Lambda(\xi,\theta)$: *functions that specify magnetic field geometry*

field lines $\Rightarrow q = const \Rightarrow \theta = F(\xi)$

Dipole Coordinate System

$$\vec{B} = B_0\left[\xi^{-3}\left(3\cos\theta\ \hat{r} - \hat{z}\right)\right] \quad \textit{where} \quad \xi = r/R_*$$

$$p = -\xi^{-2}\cos\theta \quad \textit{and} \quad q = \xi^{-1}\sin^2\theta$$

$$\left.\begin{aligned} \nabla p &= 2\xi^{-3}\cos\theta\ \hat{r} + \xi^{-3}\sin\theta\ \hat{\theta} \\ \nabla q &= -\xi^{-2}\sin^2\theta\ \hat{r} + 2\xi^{-2}\cos\theta\sin\theta\ \hat{\theta} \end{aligned}\right\} \quad \nabla p \bullet \nabla q = 0$$

$$h_p = \xi^3\left[4\cos^2\theta + \sin^2\theta\right]^{-1/2}$$

$$h_p = \frac{\xi^2}{\sin\theta}\left[4\cos^2\theta + \sin^2\theta\right]^{-1/2} \qquad \left(h_\phi = \xi\sin\theta\right)$$

Dipole Ancillary Functions

For Coordinate System that follows Dipole Magnetic Field Lines:

$$\Upsilon = \Upsilon(\xi) = \frac{3}{2}\;\frac{8 - 5q\xi}{4 - 3q\xi}$$

$$\Lambda = \Lambda(\xi) = \frac{3}{2} q\;\xi^2$$

along field line labeled by the coordinate q

Dipole Solutions (Isothermal)

Accretion flow follows magnetic field lines, which are lines of constant coordinate q.

$$3\xi \frac{8-5q\xi}{4-3q\xi} = 2b - 3\ \omega\ q\ \xi^4$$

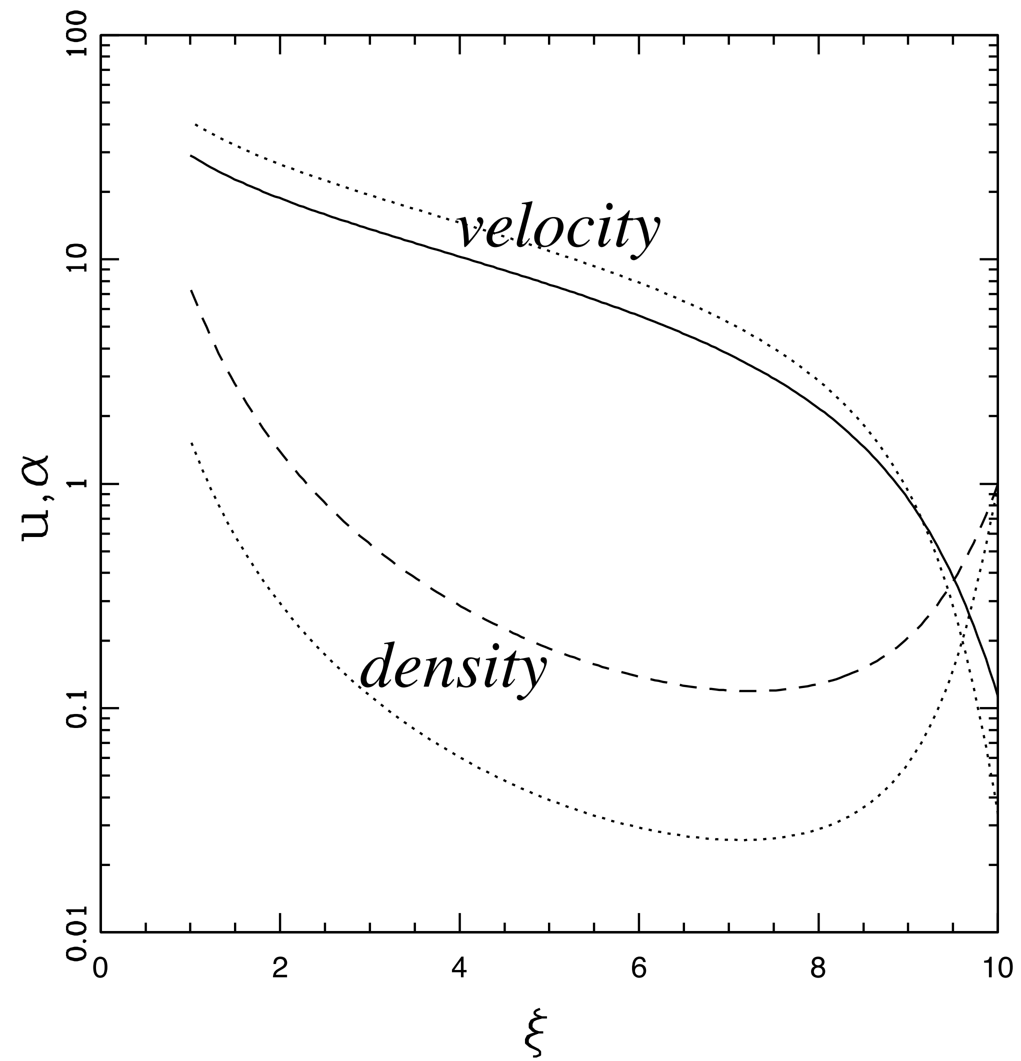
(Sonic Point Condition)

$$\log\lambda - \frac{1}{2}\lambda^2 = 3\log\xi_s - \frac{1}{2}\log(4-3\xi_s) - \frac{1}{2}$$

(Mass Accretion Rate)

$$+\,b\left(\frac{1}{\xi_s} + \frac{1}{2}\xi_s^2 - \frac{3}{2}\right)$$

Dipole Accretion Solution



dipole fields

isothermal flow

$\xi_d = 10$

$\xi_* = 1$

$b = 500$

$b = 1000$ (*dots*)

General Constraint on Steady Polytropic Transonic Accretion Flow

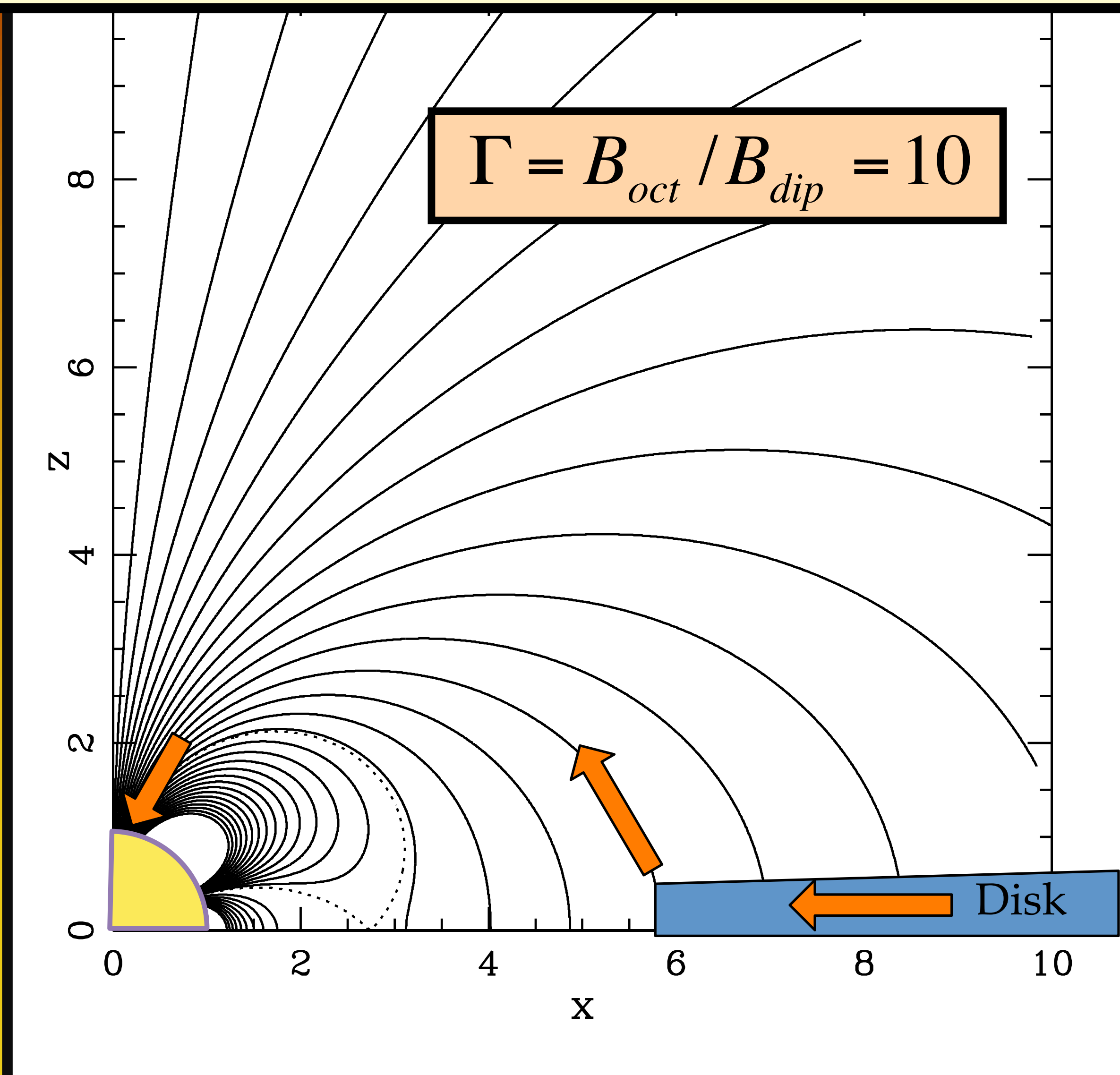
Observations show that flow must be ***transonic***. ***Steady-state*** accretion solutions that pass ***through the sonic point*** and approach ***free-fall speed*** near the star must satisfy the constraint:

$$n > \ell + 3/2$$

$$\rightarrow \; n > 9/2 \;\; (octupole)$$

Steady flow must be nearly isothermal for fields with higher order multipoles.

Dipole + Octupole Configuration



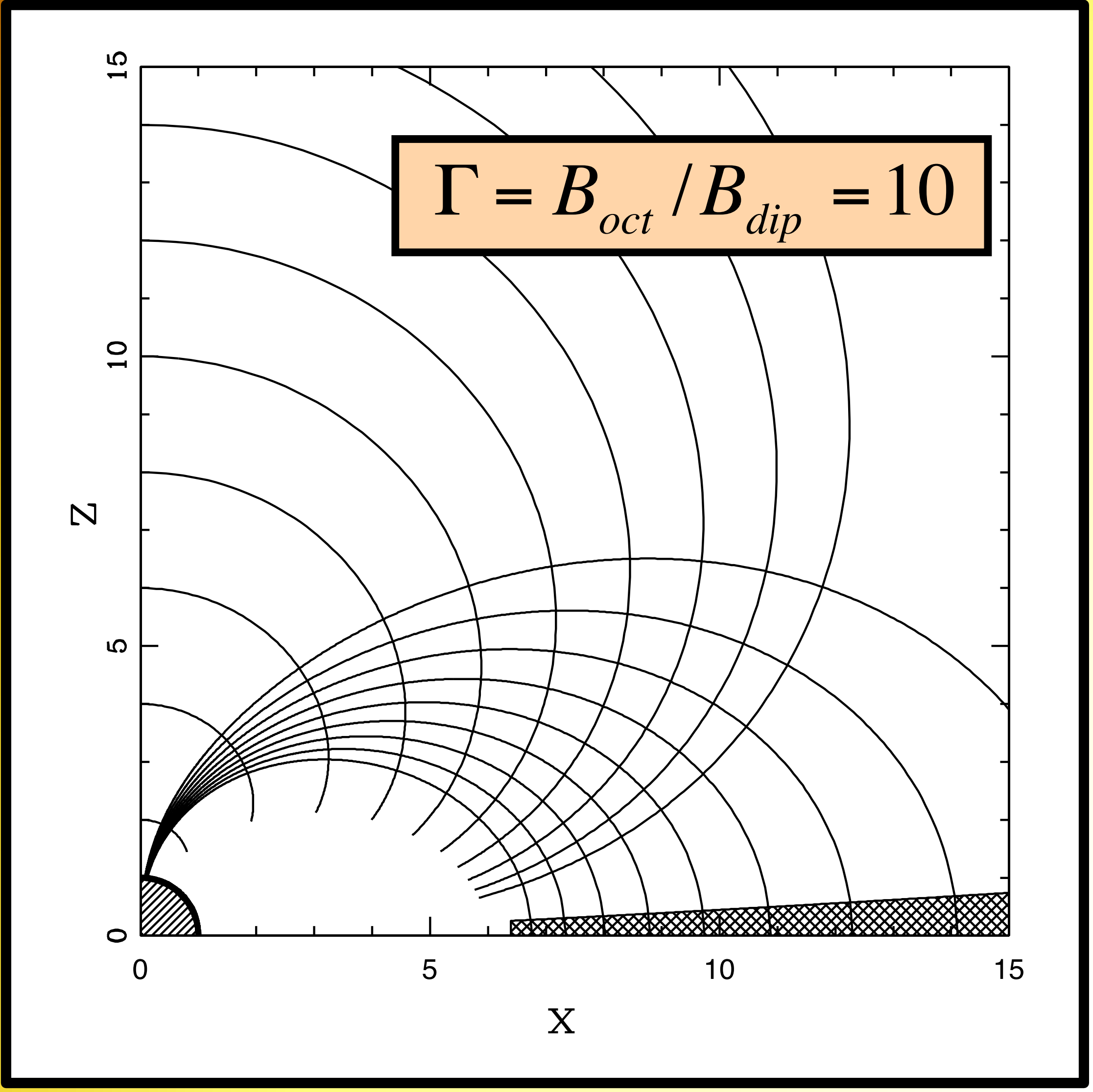
Dipole + Octupole Coordinate System

$$\vec{B} = B_{dip}\xi^{-3}\left(2\cos\theta\ \hat{r} + \sin\theta\ \hat{\theta}\right)\ +$$

$$\frac{1}{2}B_{oct}\xi^{-5}\left[\left(5\cos^2\theta - 3\right)\cos\theta\ \hat{r} + \frac{3}{4}\left(5\cos^2\theta - 1\right)\sin\theta\ \hat{\theta}\right]$$

$$\left.\begin{aligned} p &= -\frac{\Gamma}{4}\xi^{-4}\left(5\cos^2\theta - 3\right)\cos\theta - \xi^{-2}\cos\theta \\ q &= \frac{\Gamma}{4}\xi^{-3}\left(5\cos^2\theta - 1\right)\sin^2\theta + \xi^{-1}\sin^2\theta \end{aligned}\right\}\ \textit{where}\quad \Gamma \equiv \frac{B_{oct}}{B_{dip}}$$

Dipole + Octupole Coordinate System



Dipole + Octupole Scale Factors

$$h_p = \xi^5 \; [f^2 \cos^2\theta + g^2 \sin^2\theta]^{-1/2}$$

$$h_q = \xi^4 \; (\sin\theta)^{-1} \; [f^2 \cos^2\theta + g^2 \sin^2\theta]^{-1/2}$$

$$where \quad f = \Gamma \, (5\cos^2\theta - 3) + 2\xi^2$$

$$and \quad g = (3/4)\Gamma \, (5\cos^2\theta - 1) + \xi^2$$

$$\sin^2\theta = \frac{2}{5\Gamma}\left\{(\xi^2 + \Gamma) - \left[(\xi^2 + \Gamma)^2 - 5\Gamma q \xi^3\right]^{1/2}\right\}$$

$$so \;\; that \quad f, \; g, \; h_p, \; h_q = Function(\xi \; only)$$

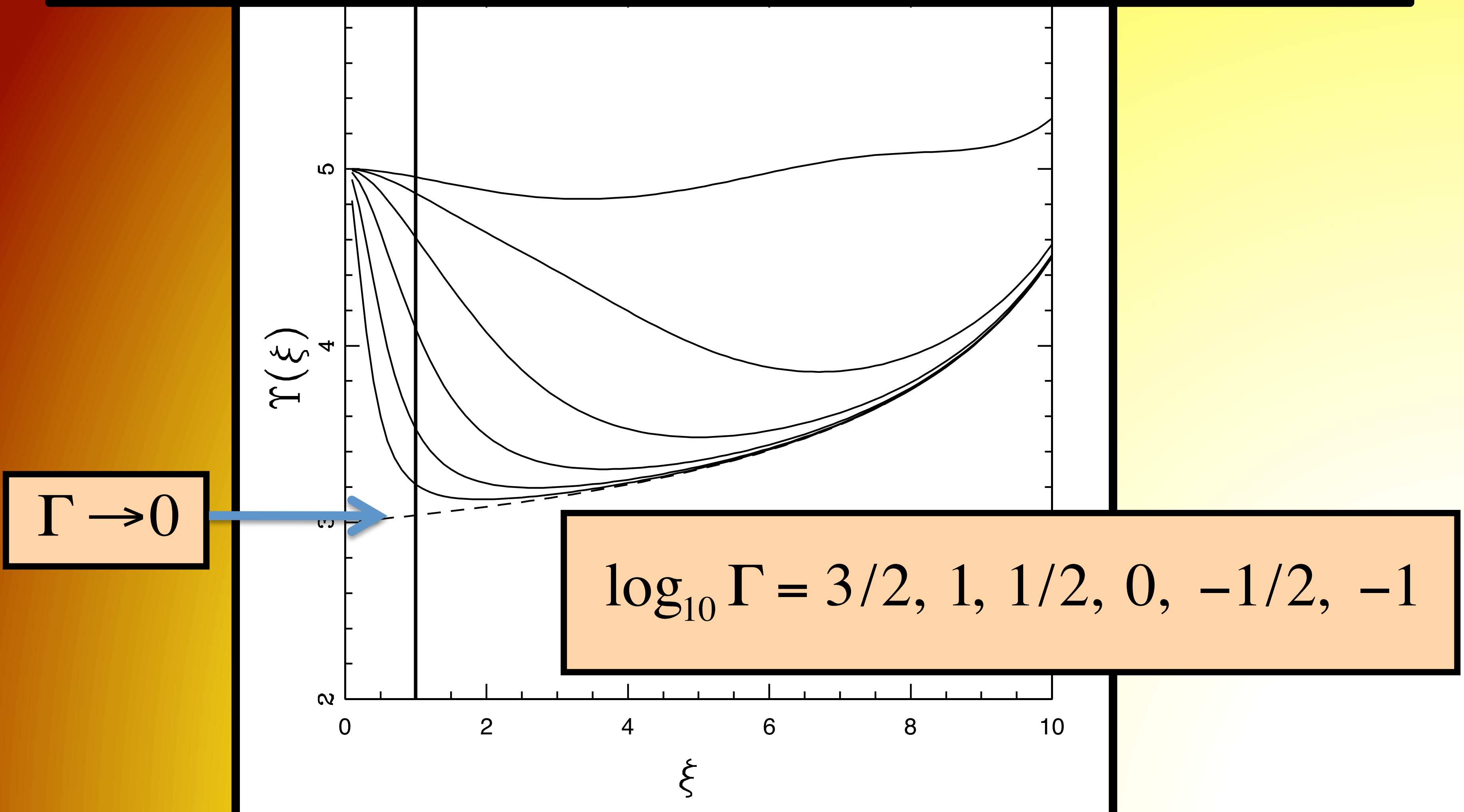
Ancillary Functions for Dipole + Octupole Configuration

$$\Lambda(\xi) = \frac{3\xi}{5\Gamma}\left\{1+\frac{g(\xi)}{f(\xi)}\right\}\left\{(\xi^2+\Gamma)-\left[(\xi^2+\Gamma)^2-5\Gamma q\xi^3\right]^{1/2}\right\}$$

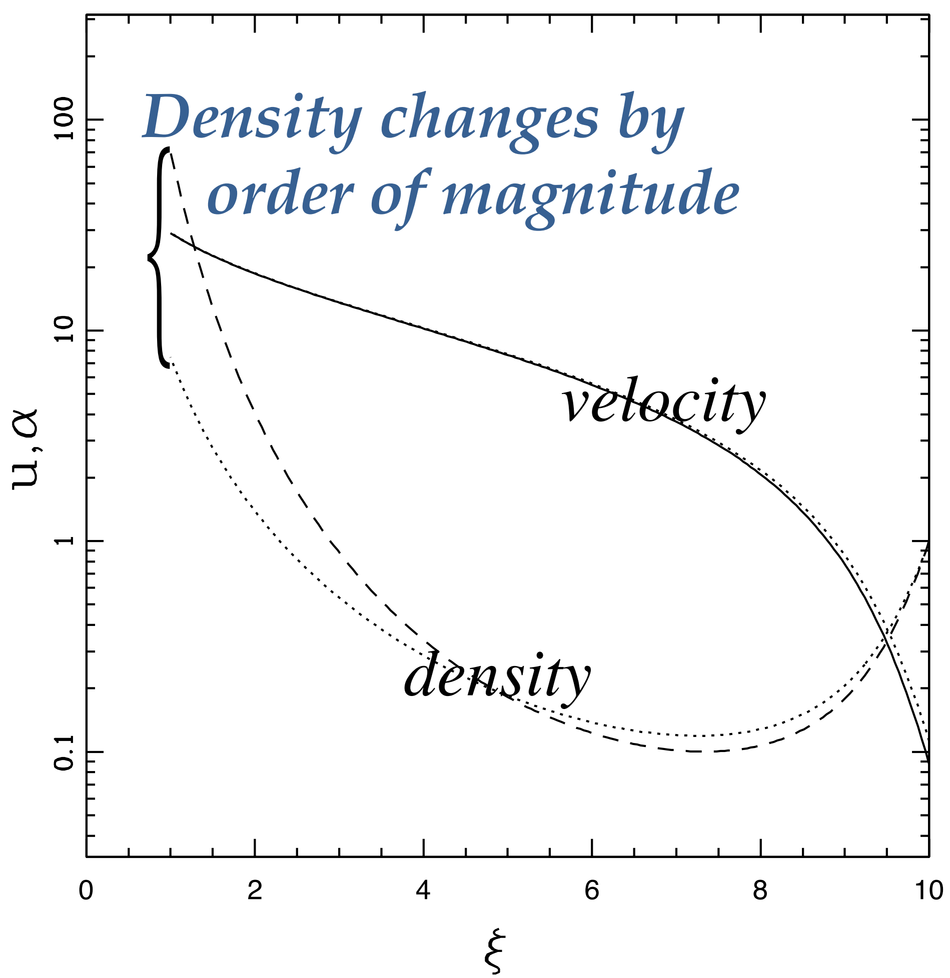
$$\Upsilon(\xi) = 5-\left[f^2+(g^2-f^2)\frac{1}{5\Gamma}(2\xi^2+2\Gamma-f)\right]^{-1}\frac{\xi}{5\Gamma}\times$$

$$\left\{5\Gamma f\, f_\xi+\left[g\left(\frac{3f_\xi}{4}-\xi\right)-f\, f_\xi\right](2\xi^2+2\Gamma-f)+(g^2-f^2)\left(2\xi-\frac{f_\xi}{2}\right)\right\}$$

Index of Divergence Operator (Dipole + Octupole)



Accretion Solution (Dip+Oct)



isothermal flow

octupole $\Gamma = 10$

$\xi_d = 10$

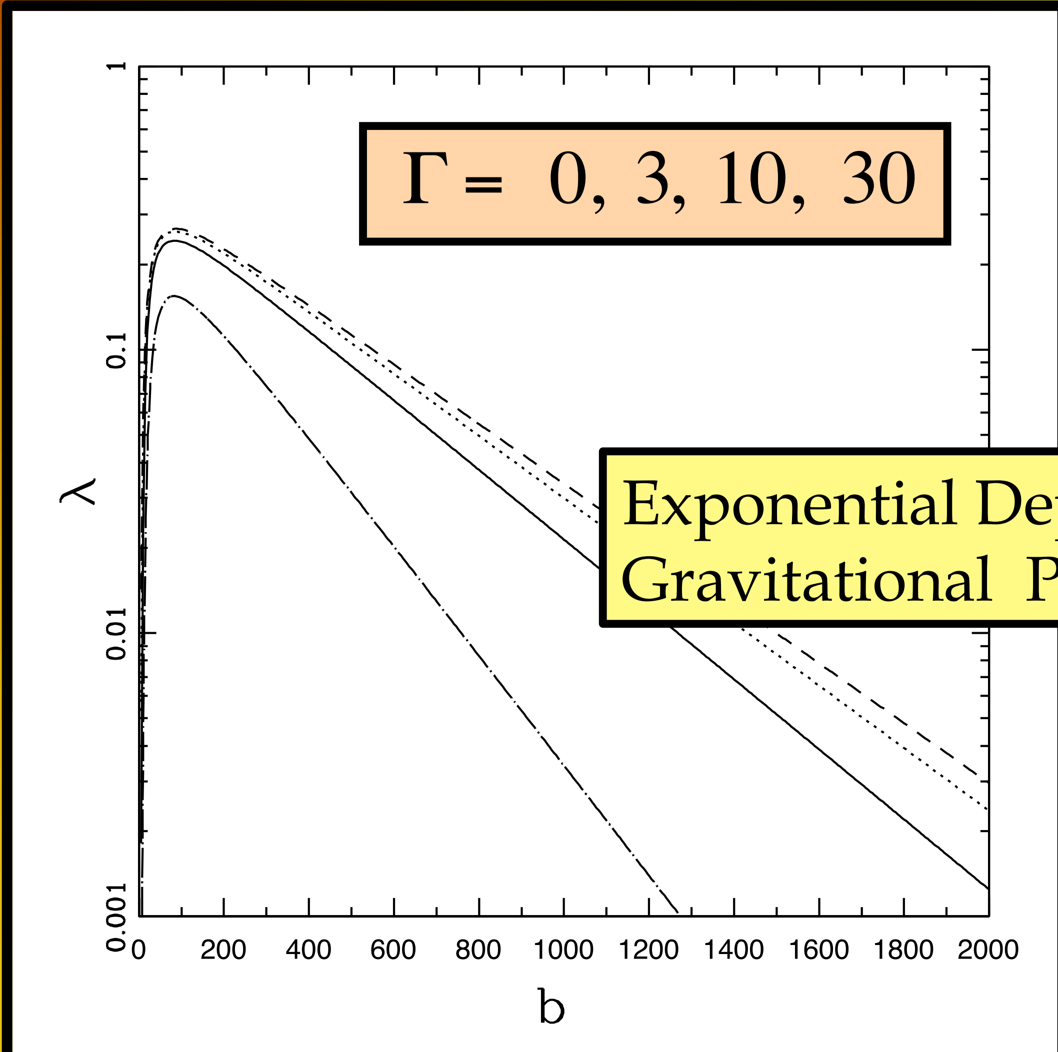
$\xi_* = 1$

$b = 500$

dots = dipole solution

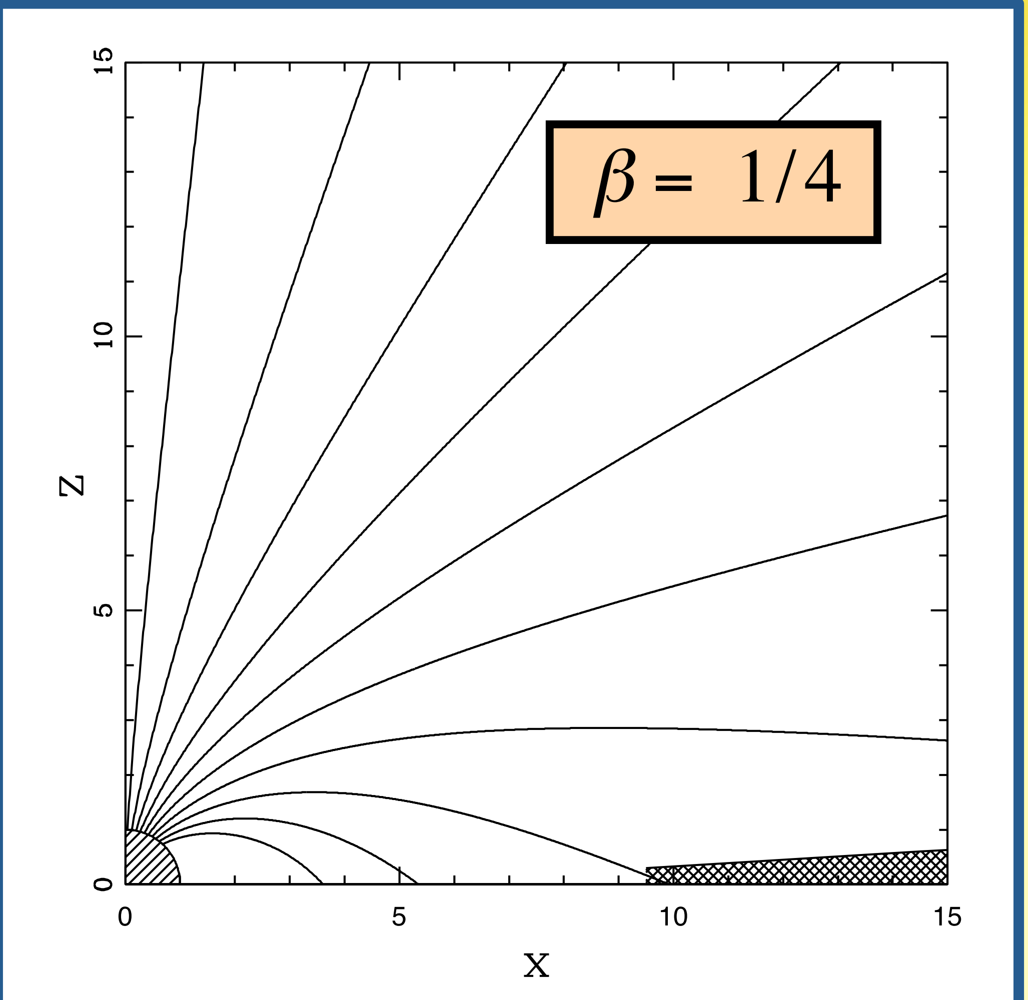
dashes = full solution

Dimensionless Accretion Rate



Exponential Dependence on Gravitational Potential b

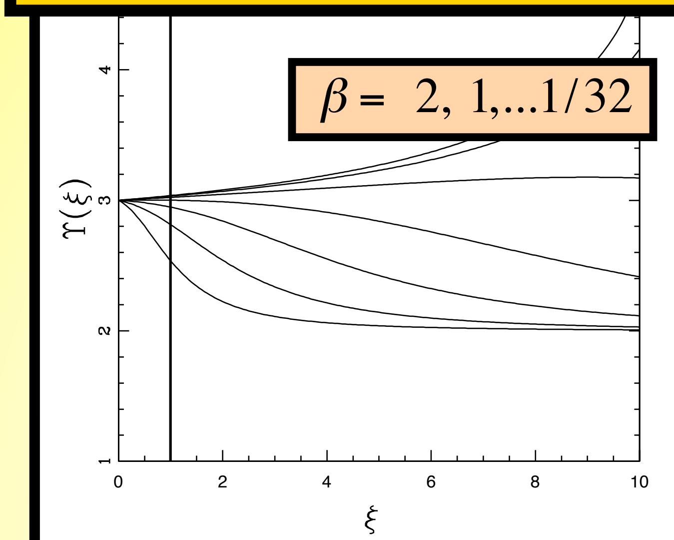
Dipole + Split-Monopole



$$p = -\xi^{-2}\cos\theta - \beta\xi^{-1}$$

$$q = \xi^{-1}\sin^2\theta - \beta\cos\theta$$

$$where \quad \beta \equiv 2B_{rad}/B_{dip}$$



Summary 2.0

- Construction of coordinate systems (p,q)
- Generalizes to many astrophysical problems
- Can find sonic points and dimensionless mass accretion rates analytically
- Dipole + Octupole system: flow density (10x)larger, hot spot has higher temperature
- Magnetic truncation radius changes
- General constraint on steady transonic flow:

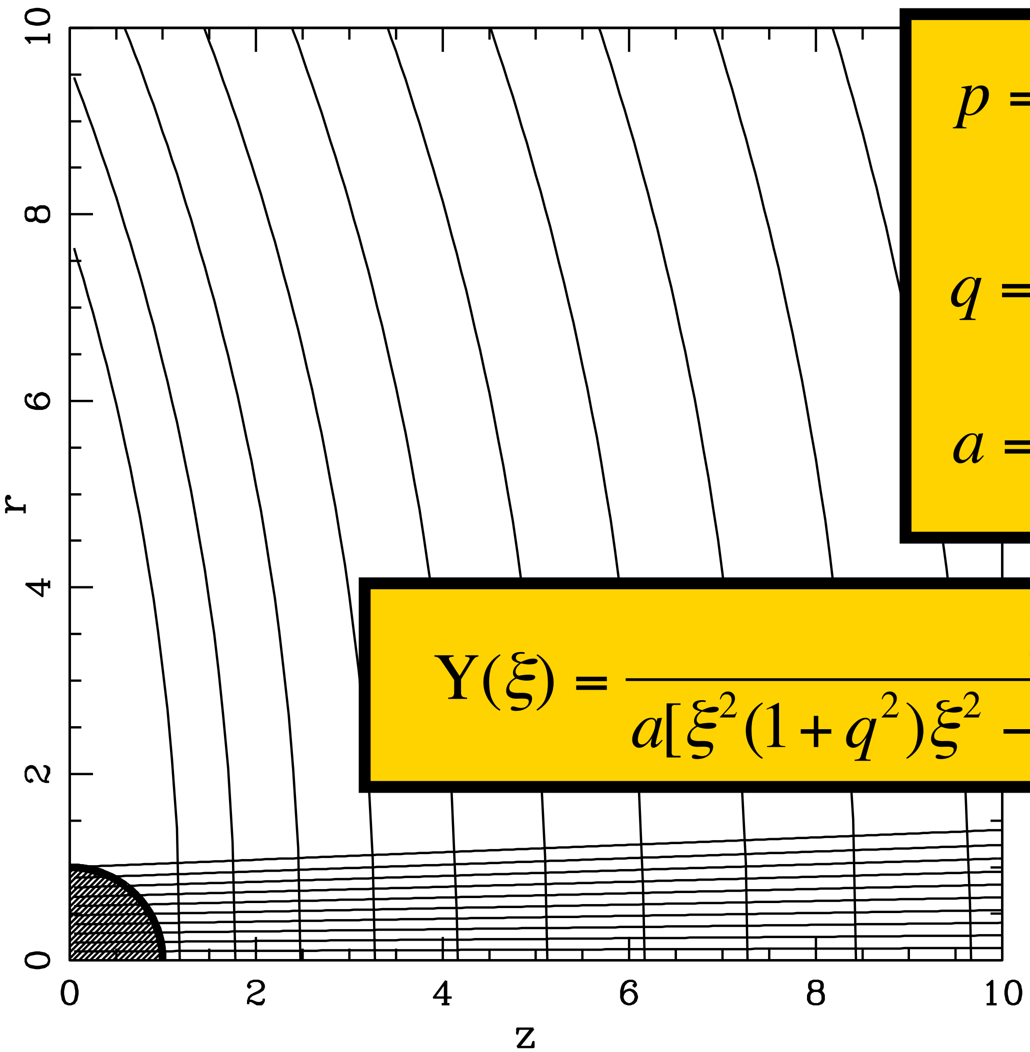
$$n > \ell + 3/2 \;\Rightarrow\; nearly \quad isothermal$$

(Adams & Gregory, 2012, ApJ, 744, 55)

Dragons

- *This use of coordinate systems in this context only works for potential fields (no currents in the flow region)*
- *The formalism has been developed for two-dimensional systems; can work for three-dimensional systems in principal, but complicated in practice*
- *Treatment (thus far) limited to steady-state (time independent) magnetic fields: magnetostatics not MHD*

Planet in Stellar Split-Monopole Field



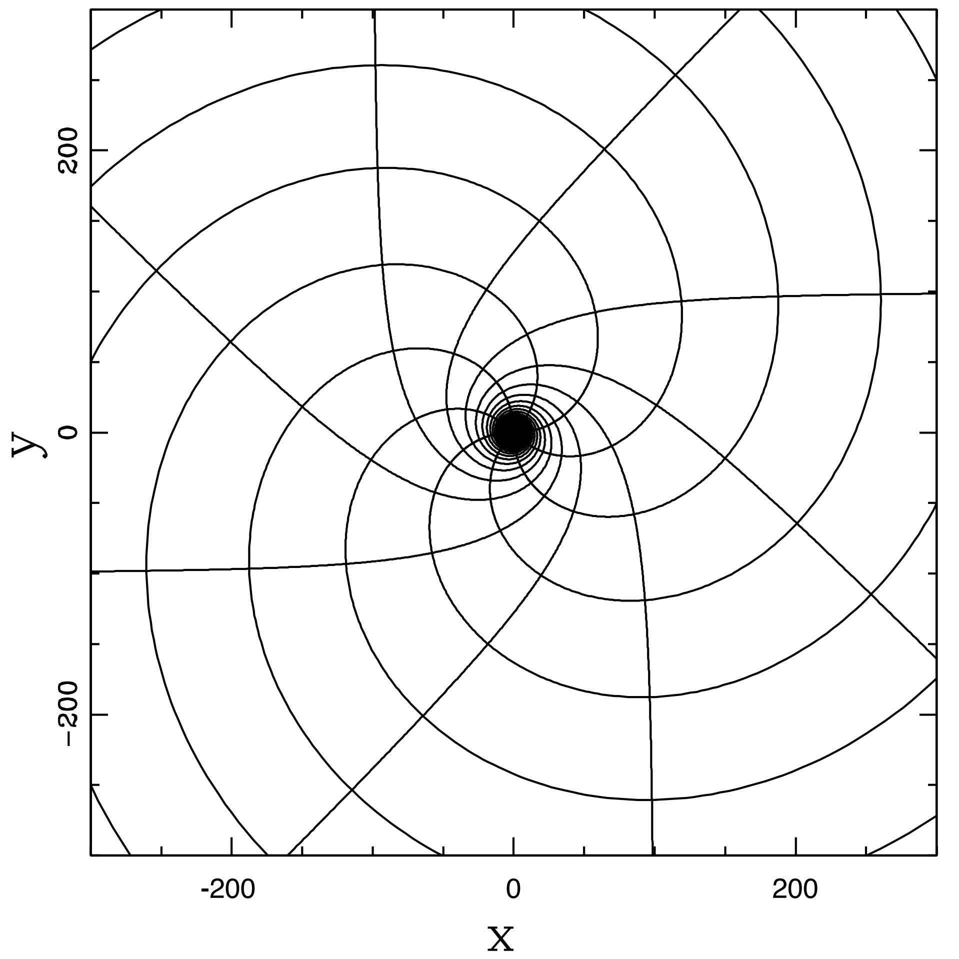
$$p = -\left(a^2 + \xi^2 + 2a\xi\cos\theta\right)^{-1/2}$$

$$q = \frac{\xi\sin\theta}{a + \xi\cos\theta}$$

$$a = radius\ of\ \ orbit$$

$$\Upsilon(\xi) = \frac{2\xi^2(1+q^2)}{a[\xi^2(1+q^2)\xi^2 - a^2q^2]^{1/2} + [(1+q^2) - a^2q^2]}$$

Parker Spiral in Equatorial Plane



$$p = A\log\left(1-\frac{1}{\xi}\right) + \phi$$

$$q = \xi - 1 - \log\xi - A\phi$$

$$A \equiv V_{wind}/(\omega R_*) \approx 100$$

(A *sets shape of spiral*)