Muon-Electron Conversion and Neutrino Magnetic Moment in Effective Theory

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Abstract

Lepton flavor violating (LFV) phenomena, such as μ-e conversion, neutrino magnetic moment (NMM), $\mu \to e\gamma$ decay, etc., are very sensitive to possible physics beyond the Standard Model (SM). Using effective field theory we parametrize new physics signatures in LFV phenomena, and get bounds for particular models, e.g., ones with leptoquarks (LQ).

Motivation

LFV is forbidden in the SM (without neutrino masses) by flavor symmetry. However it may occur at either tree or loop level in many SM extensions, which provides important clues to test them. Effective theory is a general framework for description of the effects of all these models.

In wide class of models μ-e conversion in nuclei is enhanced by large logarithms comparing with $\mu \to e\gamma$ [1].

Present bound on μ-e conversion ratio is [2]

$$R_{\mu e}^{\text{Au}} \equiv \frac{\Gamma(\mu \to e)}{\Gamma_{\text{capture}}} < 7 \times 10^{-13}, \tag{1}$$

while Mu2e sensitivity goal is 5×10^{-17} [3].

The strongest present limit on NMM [4]

$$\mu_\nu < 3 \times 10^{-12}\, \mu_B, \tag{2}$$

where $\mu_B = e/(2m_e) = 5.788... \times 10^{-5}$ eV T^{-1} is Bohr magneton, was obtained from the constraint on energy loss from globular-cluster red giants, which can be cooled faster by the plasmon decays due to NMM [5] that delays the helium ignition. The best present laboratory constraint on NMM, derived in GEMMA experiment [6], is one order of magnitude weaker than Eq. (2).

Such high sensitivity of present experiments on μ-e conversion and NMM makes them important in the new physics searches.

In the SM, minimally extended to include neutrino masses, the diagonal and transition NMMs (see Refs. in [7] and [8]),

$$\mu_{ii}^{\text{SM}} \approx 3.2 \times 10^{-20} \left(\frac{m_i}{0.1\text{ eV}}\right) \mu_B \tag{3}$$

and

$$|\mu_{ij}^{\text{SM}}| \lesssim 4 \times 10^{-24} \left(\frac{m_i + m_j}{0.1\text{ eV}}\right) \mu_B, \tag{4}$$

respectively, are strongly suppressed by the left-handed nature of the weak interaction and small masses of observable neutrinos [9]. Hence the limit in Eq. (2) leaves huge window for physics beyond the SM.

Muon-electron conversion

μ-e conversion can be described by the effective theory with the operators of dimension 5 and 6 [1]. However the operators of dim.7, involving two gluonic tensors [10], may be important since gluonic couplings to nuclei are not suppressed, and the new physics couplings to heavy quarks are not well constrained. We consider the flavor changing Lagrangian

$$\mathcal{L}_{e\mu}^{gg} = \frac{1}{\Lambda^2}\sum_{i=1}^{8} c_i O_i + \text{H.c.}, \tag{5}$$

where Λ is a scale responsible for μ-number nonconservation, c_i are coefficients of dimension -1, and O_i are effective dim.7 operators:

$$O_1 = \bar{e}_R \mu_L \tfrac{\alpha_s}{4\pi} G^a_{\mu\nu} G^{a\mu\nu}, \tag{6}$$
$$O_2 = \bar{e}_R \mu_L \tfrac{\alpha_s}{4\pi} G^a_{\mu\nu} \widetilde{G}^{a\mu\nu}, \tag{7}$$
$$O_3 = \bar{e}_L \mu_R \tfrac{\alpha_s}{4\pi} G^a_{\mu\nu} G^{a\mu\nu}, \tag{8}$$
$$O_4 = \bar{e}_L \mu_R \tfrac{\alpha_s}{4\pi} G^a_{\mu\nu} \widetilde{G}^{a\mu\nu}, \tag{9}$$

where $a = 1, \dots, 8$ is gluon color index, $\alpha_s = g_s^2/(4\pi)$, the gluon strength tensor is

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu, \tag{10}$$

and the dual one is

$$\widetilde{G}^a_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta} G^{a\alpha\beta}. \tag{11}$$

Using the diagrams with quarks propagating in loop, shown in **Fig.** 1, the Lagrangian in Eq. (5) can be matched to dim.6 4-fremion scalar Lagrangian

$$\mathcal{L}_{e\mu}^{qq} = \frac{Y_{\mu i} Y_{ej}}{\Lambda^2}(\bar{e} P_\alpha \mu)(\bar{q}^i P_\beta q^j) + \text{H.c.}, \tag{12}$$

where Y are real couplings, and P stands for a projector with $\alpha, \beta = L, R$.

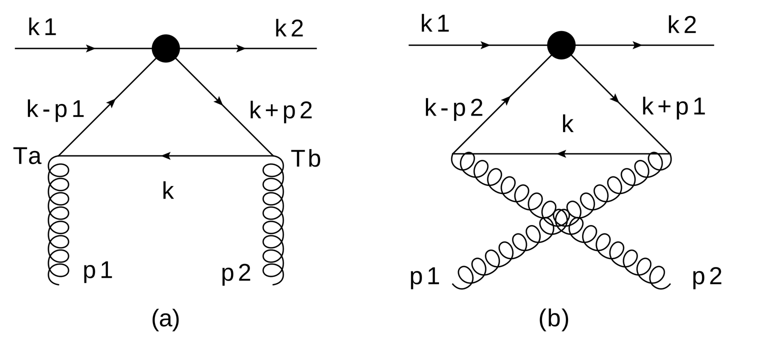

Figure 1: Discussed effective diagrams

The pseudoscalar nucleon current couples to the nuclear spin leading to incoherent ($N \neq N'$) contribution [11]. The coherent μ-e conversion rate on nucleus N can be written as

$$\Gamma_{\text{conv}}(\mu N \to eN) = \frac{4}{\Lambda^4}\left(|c_1|^2 + |c_3|^2\right) a^2, \tag{13}$$

where

$$a = G^{(g,p)} S^{(p)} + G^{(g,n)} S^{(n)} \tag{14}$$

with the overlap integrals S defined in [12], and the matrix element

$$G^{(g,\mathcal{N})} = \langle \mathcal{N} | \frac{\alpha_s}{4\pi} G^a_{\mu\nu} G^{a\mu\nu} | \mathcal{N} \rangle \tag{15}$$

with $\mathcal{N} = n, p$. For the strange-quark sigma term $\sigma_s \equiv m_s \langle p|\bar{q}q|p\rangle = 50$ MeV the numerical result is $G^{(g,\mathcal{N})} = -189$ MeV [13].

The upper bounds on the relevant parameters of the Lagrangian in Eq. (5) for one nonzero c_i at a time and for the two chosen nuclei are given in Table 1, where $i = 1, 3$.

Table 1: Bounds on the parameters in Eq. (5)

$\frac{\lvert c \rvert}{\Lambda^2}$	Expression for the bound	Bound, MeV^{-3}	
		$^{48}_{22}$Ti	$^{197}_{79}$Au
$\frac{\lvert c_i \rvert}{\Lambda^2}$	$\frac{\Gamma^{1/2}_{\text{conv}}(\mu N \to eN)}{2\lvert a_N \rvert}$	2.46×10^{-20}	1.17×10^{-20}

Models with leptoquarks

The bounds in Table 1 can be used for restricting the physical parameters of particular models, e.g., models with LQs. The general renormalizable, B and L conserving, and $SU(3)\times SU(2)\times U(1)$ invariant LQ-lepton-quark interactions are given in Refs. [14, 15, 16]. The scalar (S) and vector (V) LQ interactions relevant for the Lagrangian in Eq. (5) are

$$\begin{aligned}\mathcal{L}_S &= \left(\lambda_{LS_0} \bar{q}^c_L \mathrm{i}\tau_2 \ell_L + \lambda_{RS_0} \bar{u}^c_R e_R\right) S_0^\dagger \\ &+ \left(\lambda_{LS_{1/2}} \bar{u}_R \ell_L + \lambda_{RS_{1/2}} \bar{q}_L \mathrm{i}\tau_2 e_R\right) S_{1/2}^\dagger + \text{H.c.}, \\ \mathcal{L}_V &= \left(\lambda_{LV_0} \bar{q}_L \gamma_\mu \ell_L + \lambda_{RV_0} \bar{d}_R \gamma_\mu e_R\right) V_0^{\mu\dagger} \\ &+ \left(\lambda_{LV_{1/2}} \bar{d}^c_R \gamma_\mu \ell_L + \lambda_{RV_{1/2}} \bar{q}^c_L \gamma_\mu e_R\right) V_{1/2}^{\mu\dagger} + \text{H.c.},\end{aligned} \tag{16}$$

where we omit flavor indeces, the subindexes 0 and 1/2 indicate $SU(2)$ singlet and doublet LQ, respectively; and couplings λ are assumed to be real. These LQ interactions induce the effective vertices of the form of Eq. (12). By further matching with Eq. (5), assuming that only the couplings λ for a single quark flavor are nonzero at a time, for the common scales M_S and M_V of scalar and vector LQ masses, respectively, from the bound on μ-e conversion on gold we have

$$\begin{aligned}|\lambda^\alpha_{RS_0}\lambda^\beta_{LS_0}| &= |\lambda^\alpha_{RS_{1/2}}\lambda^\beta_{LS_{1/2}}| \\ &< 1.2 \times 10^{-2} \left(\frac{M_S}{1\text{ TeV}}\right)^2,\end{aligned} \tag{17}$$

$$\begin{aligned}|\lambda^\alpha_{LV_0}\lambda^\beta_{RV_0}| &= |\lambda^\alpha_{LV_{1/2}}\lambda^\beta_{RV_{1/2}}| \\ &< 1.6 \times 10^{-4} \left(\frac{M_V}{1\text{ TeV}}\right)^2,\end{aligned} \tag{18}$$

where $\alpha \neq \beta = e, \mu$.

Neutrino magnetic moment

Similar approach can be applied for NMM $\mu^\nu_{\alpha\beta}$, which is defined by the form factor

$$f^M_{\alpha\beta}(0) = \mu^\nu_{\alpha\beta} \tag{19}$$

of the term in the effective neutrino current

$$-f^M_{\alpha\beta}(q^2)\, \bar{\nu}_\beta(p_2)\, i\sigma_{\mu\nu} q^\nu \nu_\alpha(p_1), \tag{20}$$

where $q = p_2 - p_1$, and $\sigma_{\mu\nu} = i[\gamma_\mu, \gamma_\nu]/2$.

NMM generically induces a radiative correction to the neutrino mass, which constrains NMM [7, 17, 18]. In the case of diagonal NMM (Dirac neutrinos) the correspondent bound $\mu_{\alpha\alpha} \lesssim 10^{-14}\, \mu_B$ is significantly stronger than in Eq. (2). However, the transition NMM $\mu_{\alpha\beta}$, which is possible for both Dirac and Majorana neutrino types, is antisymmetric in the flavor indices, while the neutrino mass terms $m^\nu_{\alpha\beta}$ are symmetric. This may lead to suppression of the $\mu_{\alpha\beta}$ contribution to $m^\nu_{\alpha\beta}$, e.g., by the SM Yukawas, which makes the bound on NMM much weaker than in Eq. (2): $\mu_{\alpha\beta} \lesssim 10^{-9}\, \mu_B$ [7, 18].

In the general quark-ν dim.6 Lagrangian [19]

$$\mathcal{L}^{\nu q}_{\text{eff}} = \frac{\epsilon^{q\Gamma}_{\alpha\beta}}{M^2}(\bar{\nu}_\beta \Gamma \nu_\alpha)(\bar{q}\Gamma q) + \text{H.c.}, \tag{21}$$

where M is high-energy scale, only the term

$$\frac{\epsilon^q_{\alpha\beta}}{M^2}(\bar{\nu}_\beta \sigma_{\mu\nu} \nu_\alpha)(\bar{q}\sigma^{\mu\nu} q), \tag{22}$$

where $\epsilon^q_{\alpha\beta} \equiv \epsilon^{qT}_{\alpha\beta}$, generates through the one-loop diagram, shown in **Fig.** 2, the lowest order contribution to NMM

$$|\mu_{\alpha\beta}| = |\epsilon^q_{\alpha\beta}| \frac{N_c |Q_q|}{\pi^2} \frac{m_e m_q}{M^2} \ln\left(\frac{M^2}{m_q^2}\right) \mu_B, \tag{23}$$

where $N_c = 3$ is the number of colors, Q_q and m_q are quark charge and mass, respectively; and we neglected the subleading term, which is not enhanced by the large logarithm.

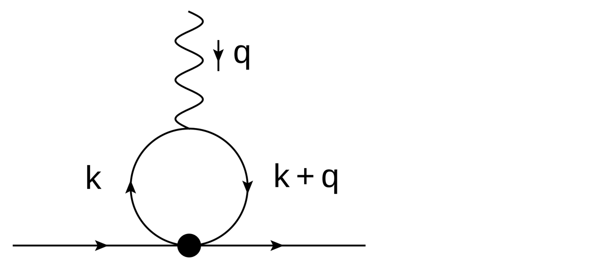

Figure 2: Effective diagram for NMM

And for the tensor term in the charge lepton-ν dim.6 Lagrangian [19, 20, 21]

$$\mathcal{L}^{\nu\ell}_{\text{eff}} = \frac{\epsilon^{\ell\Gamma}_{\alpha\beta}}{M^2}(\bar{\nu}_\beta \Gamma \nu_\alpha)(\bar{\ell}\Gamma \ell) + \text{H.c.}, \tag{24}$$

denoting $\epsilon^\ell_{\alpha\beta} \equiv \epsilon^{\ell T}_{\alpha\beta}$, we have

$$|\mu_{\alpha\beta}| = \frac{|\epsilon^\ell_{\alpha\beta}|}{\pi^2} \frac{m_e m_\ell}{M^2} \ln\left(\frac{M^2}{m_\ell^2}\right) \mu_B. \tag{25}$$

For $M = 1$ TeV, using Eq. (2) and taking one nonzero $\epsilon^f_{\alpha\beta}$ at a time, we obtain the constraints shown in Tables 2 and 3 [22].

Table 2: Upper bounds on the couplings $\epsilon^q_{\alpha\beta}$.

$\lvert\epsilon^d_{\alpha\beta}\rvert$	0.25	$\lvert\epsilon^u_{\alpha\beta}\rvert$	0.49
$\lvert\epsilon^s_{\alpha\beta}\rvert$	1.6×10^{-2}	$\lvert\epsilon^c_{\alpha\beta}\rvert$	1.7×10^{-3}
$\lvert\epsilon^b_{\alpha\beta}\rvert$	5.8×10^{-4}	$\lvert\epsilon^t_{\alpha\beta}\rvert$	4.8×10^{-5}

Table 3: Upper bounds on the couplings $\epsilon^\ell_{\alpha\beta}$.

$\lvert\epsilon^e_{\alpha\beta}\rvert$	3.9
$\lvert\epsilon^\mu_{\alpha\beta}\rvert$	3.0×10^{-2}
$\lvert\epsilon^\tau_{\alpha\beta}\rvert$	2.6×10^{-3}

We note that Eq. (23) reproduces the leading order in the exact result, which can be derived in the model with scalar LQs; see Ref. [23] for the exact expressions of diagonal NMMs.

We note that the bound on $\epsilon^e_{e\beta}$ from $\bar{\nu}_e$–e scattering, derived in [24], is comparable with the respective bound in Table 3.

Conclusions

We have considered the muon-electron conversion in nuclei and the neutrino magnetic moment within the effective theory framework, and derived general constraints on the physics beyond the standard model, which can be involved in these phenomena.

Acknowledgments

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