Global Simulations of Magnetorotational Instability in Core-Collapse Supernovae

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MAGNETIC FIELDS IN THE UNIVERSE IV

1. Introduction

Core-collapse supernovae (CCSNe) with a strong pre-collapse B-field
→ well studied for the decade.

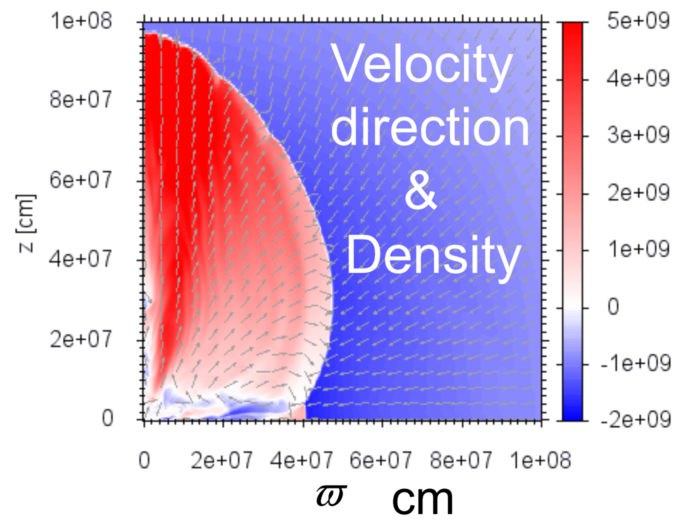
✓ The B-field amplified due to differential rotation drives explosion.
(Symbalisity 84, Yamada & Sawai 04)

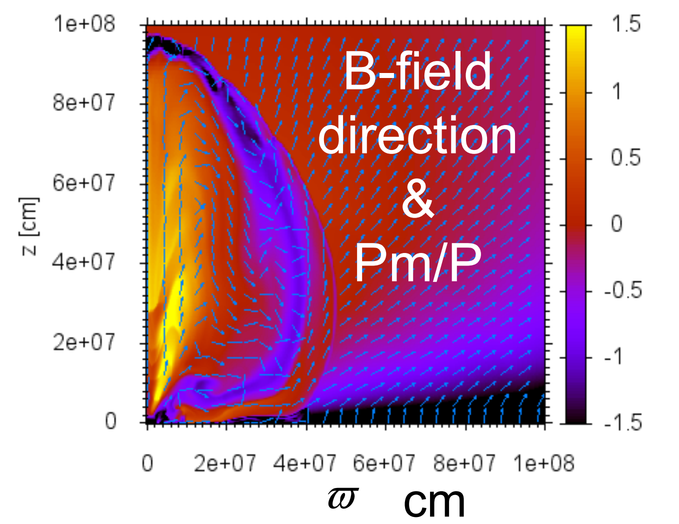
$E_{grv} \rightarrow E_{rot} \rightarrow E_{mag} \rightarrow E_{kin}$

✔ Required strength of B-field and rotation.

$B_{in} \sim 10^{13}$ G $B_{PNS} \sim 10^{15}$ G (magnetar class)

$P_{in} \sim 1$ s $P_{PNS} \sim 1$ ms





The B-field at the pre-collapse stage is uncertain.

✓Stellar evolution simulation

→ B-filed is weak (B_{pol}~ 10^6 G, B_{tor}~ 10^9 G) Heger+05

✓ Observation

→ Some OB stars posses strong (magnetar-class) B-field

O star: θOrion C, HD191612 Donati+02,06

B star: ξCMa, V2052 Oph, ωOri, ζ Cas Hubrig+06, Neiner+03

✓ Population synthesis calculation assuming fossil origin hypothesis

→ ~10% of OB stars posses strong (magnetar-class) B-field

Ferrario & Wickramasinghe 05

At present, neither weakly nor strongly magnetized progenitor is excluded.

Both possibility should be studied in CCSNe.

However, weakly magnetized progenitor is not studied enough.

MRI： Obergaulinger+09, Masada+12

Convection, SASI: Obergaulinger+11, Endeve+12

Previous numerical works on MRI in CCSNe

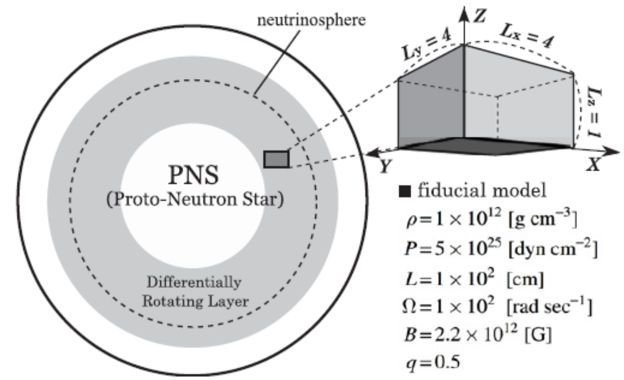
Masada+ 12

Obergaulinger+09, Masada+12 :

- 2D, 3D Local Simulation

  L ~ km, Δx ∼ 1-10 m

- $B_{PNS,0}$~ 10^{12}-10^{13} G, ρ ~ 10^{12}-10^{13} g/cc



Local Simulation

✔Advantage

- Getting high resolution.

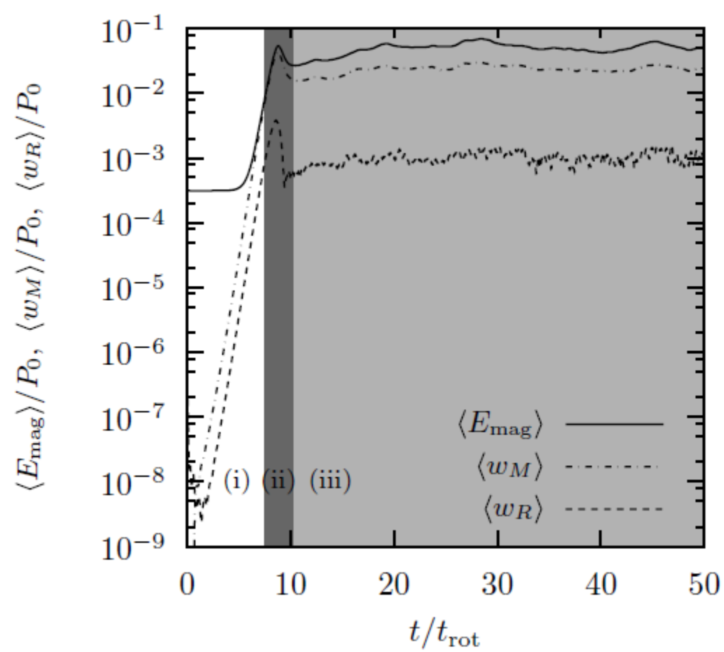
  → 3D simulation possible.

✔Drawback

- Difficulty in setting a proper background.

  ← A post-bounce core is dynamical.

- The global dynamics cannot be studied.



No global simulations so far.

This work

We carry out first Global simulations of MRI from a sub-magnetar-class B-field in CCSNe.

✔Motivation

Studying the evolution and effects of B-field in the CCSN for a weakly magnetized, rapidly rotating progenitor.

- Does MRI occur in the dynamical post-bounce core?
- Is a B-filed amplified to a magnetar-class strength?
- Does the amplified B-field affect the dynamics?

1. Numerical Method and Model

◆Resolution required for simulations of MRI

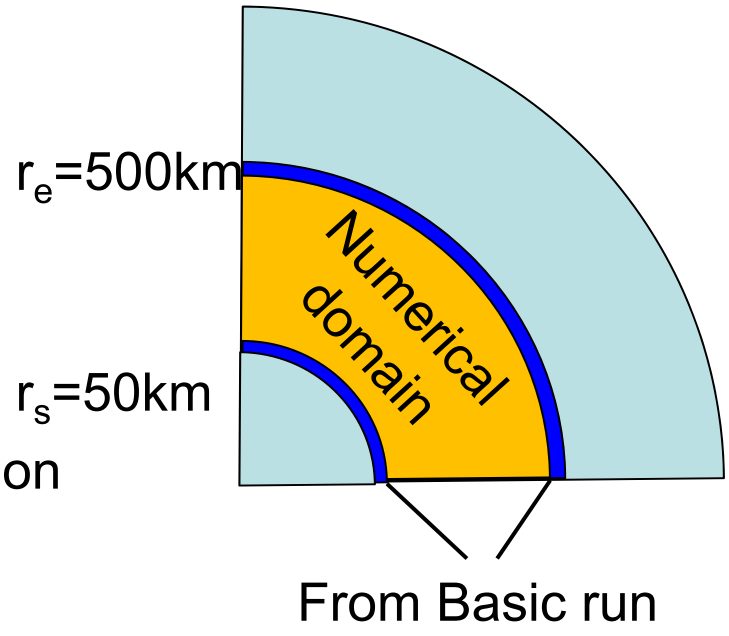
$$\lambda \sim \sqrt{\frac{\pi}{\rho}}\frac{B}{\Omega} \sim 5\times10^{4}\,cm \times \left(\frac{\rho}{10^{11}\,g/cm^{3}}\right)^{-\frac{1}{2}}\left(\frac{B}{10^{13}\,G}\right)\left(\frac{\Omega}{10^{3}\,rad/s}\right)^{-1} \Rightarrow \Delta r \sim 5\times10^{3}\,cm$$

◆Numerical domain for MRI run

A part of the core

50 < (r / km) < 500

Axisymmetry and equatorial-symmetry are assumed.



◆Initial condition and boundary condition

The collapse is first followed with low resolution inside the 4000 km radius until 100 ms after bounce (basic run).

✓The data 5ms after bounce is mapped into the numerical domain for MRI run.

✓The basic run data is used for boundary condition for MRI run.

◆Numerical code and equations

2D-resistive MHD code (Yamazakura)

- High resolution central scheme (Kurganov & Tadmor 2000)
- The 3rd order in time and 2nd order in space
- Constraint Transport scheme to assure divB=0

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\frac{\partial}{\partial t}(\rho \mathbf{v}) + \nabla \cdot \left(\rho \mathbf{v}\mathbf{v} - \frac{\mathbf{B}\mathbf{B}}{4\pi} \right) = -\nabla \left(p + \frac{B^2}{8\pi} \right) - \rho \nabla \Phi$$

$$\frac{\partial}{\partial t}\left(e + \frac{\rho v^2}{2} + \frac{B^2}{8\pi} \right) + \nabla \cdot \left[\left(e + p + \frac{\rho v^2}{2} + \frac{B^2}{4\pi} \right) \mathbf{v} - \frac{(\mathbf{v} \cdot \mathbf{B})\mathbf{B}}{4\pi} \right] = -\rho (\nabla \Phi) \cdot \mathbf{v}$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \left(-\mathbf{v} \times \mathbf{B} \right) = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial \Phi}{\partial r} \right) = 4\pi G \overline{\rho}(r)$$

Spherically symmetric
Newtonian gravitational potential

- Nuclear EOS by Shen+98
- Ye: A function of density (Liebendorfer 05)
- No treatment of neutrinos

Computed by SR16000 in YITP maximally with 2048 parallelization

◆Numerical Model

✓Progenitor: 15 Msun (Woosley '95)

✓B-field: Dipole-like

$B_{c,in} \sim 1.0\times 10^{11}$ G → B($\rho=10^{11}$g/cc) ~ 10^{13} G

(Em/W)in =5×10^{-5} %

✓Rotation: rapid, differential

Ωc,in = 3.9 rad/s

(T/W)in = 0.5 %

Differential rotation

$$\Omega = \Omega_0 \frac{r_0^2}{r_0^2 + r^2}$$

r_0=1000 km

✓Spatial resolution: 5 models

Δr_{min} = 13 m (8900×6400)

Δr_{min} = 25 m (4700×3200)

Δr_{min} = 50 m (2500×1600)

Δr_{min} = 100 m (1200× 800)

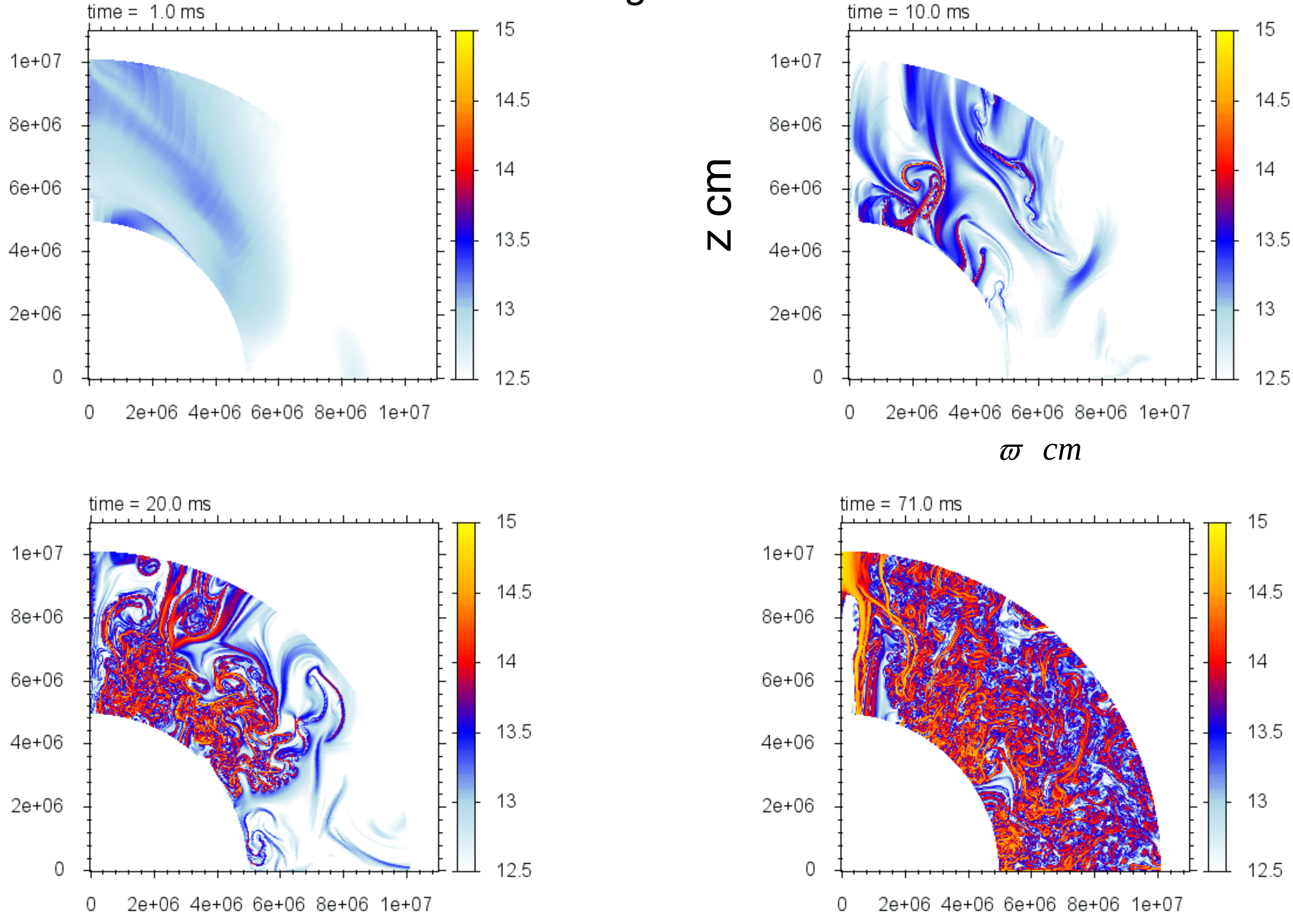
Δr_{min} = 200 m (600× 400)

3. Results

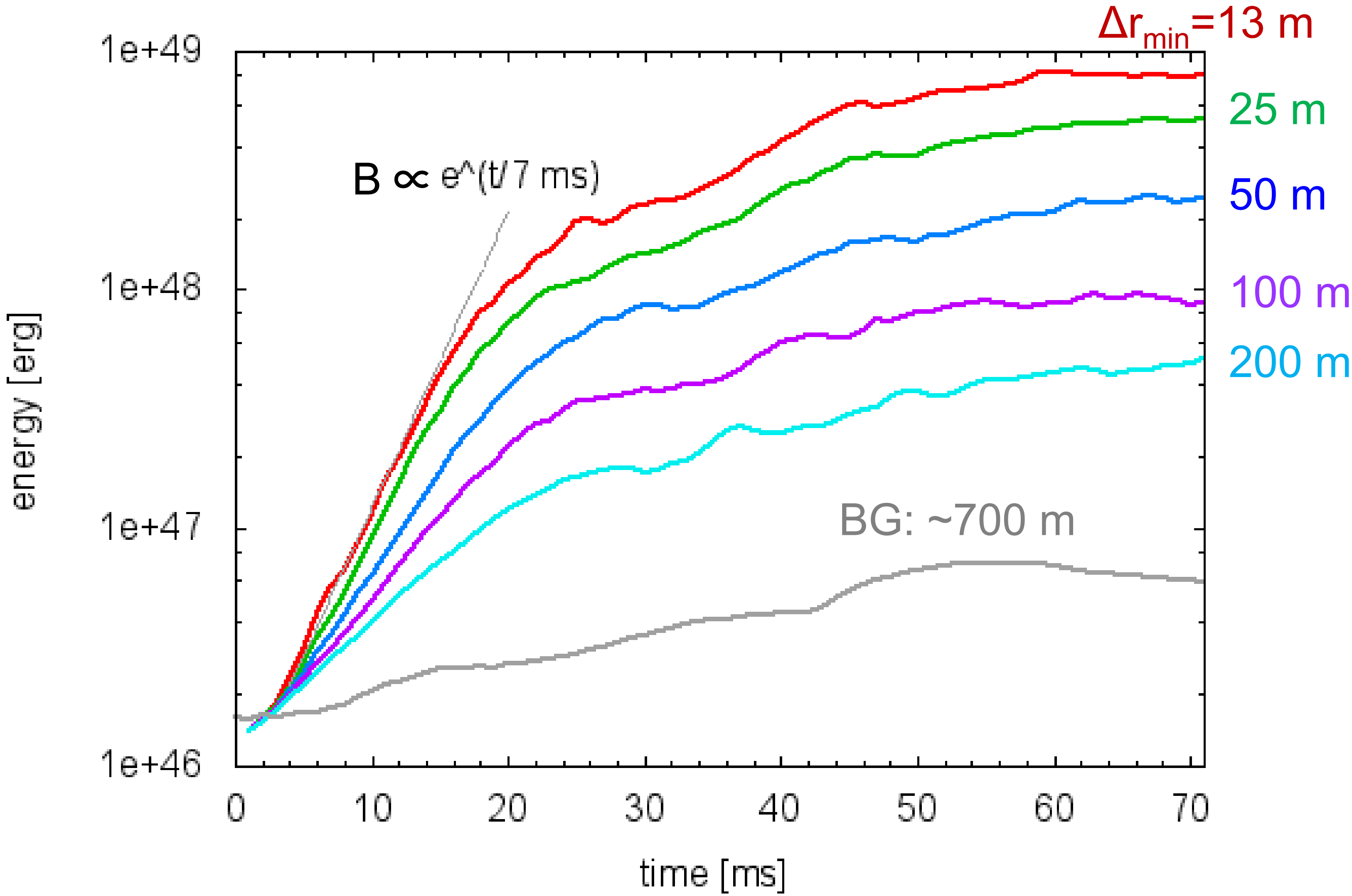
$\Delta r_{min} = 25$ m

Distribution of the poloidal B-field strength

log[Bp G]



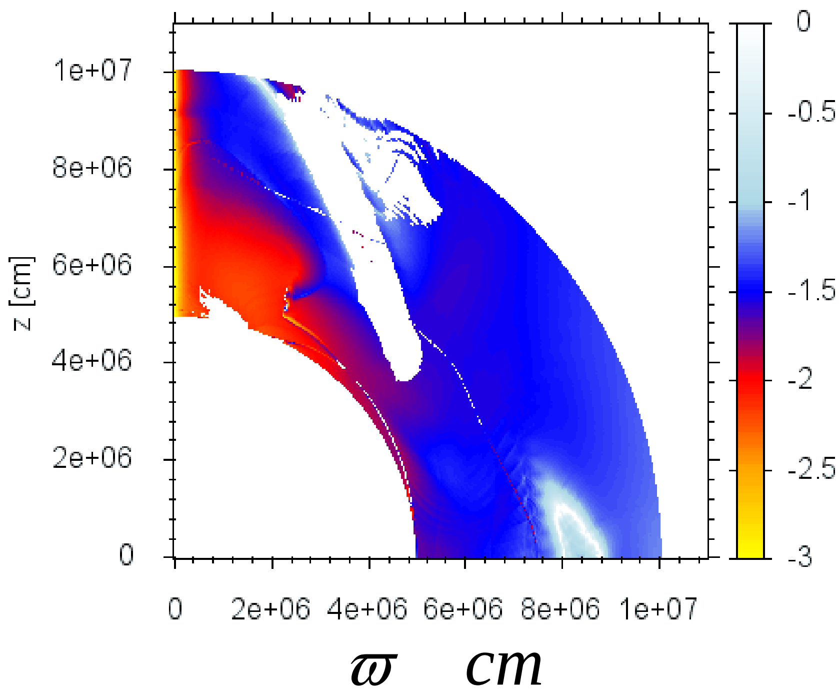
Evolution of the magnetic energy for the poloidal component of the B-field



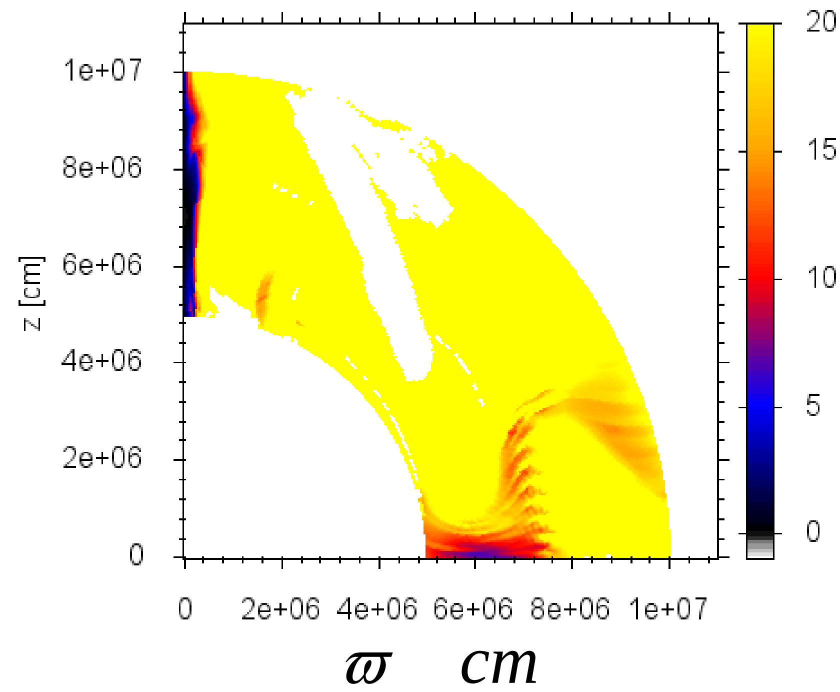
The exponential growth is observed.

Δr_{min} = 13 m   t = 4 ms

Maximum growing timescale by linear analysis



Number of grids covering maximum growing wavelength



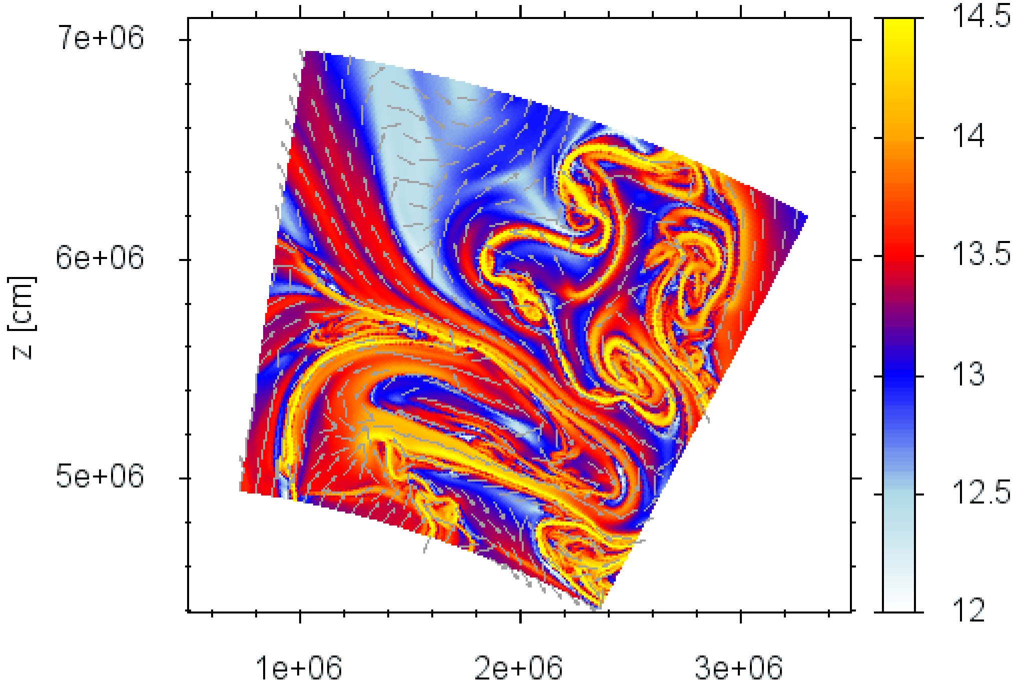
- Consistent with that MRI first occurs around the pole.
- $t_{growth,theory}$~10 ms is consistent with $t_{growth,numerical}$~7 ms.

Numerical grids are fine enough except around the pole.

Comparison with a local

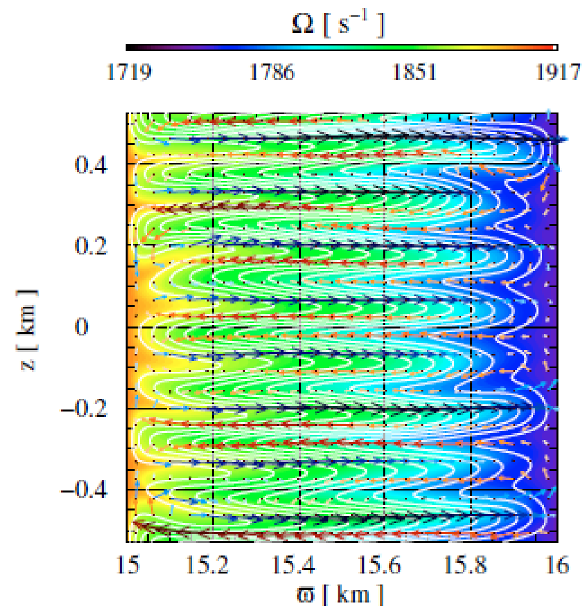
Global simulation (ours)

Δr_{min} = 13 m t = 11 ms log[G]



$\varpi \quad cm$

Local simulation in 2D without entropy gradient (Obergaulinger+09)

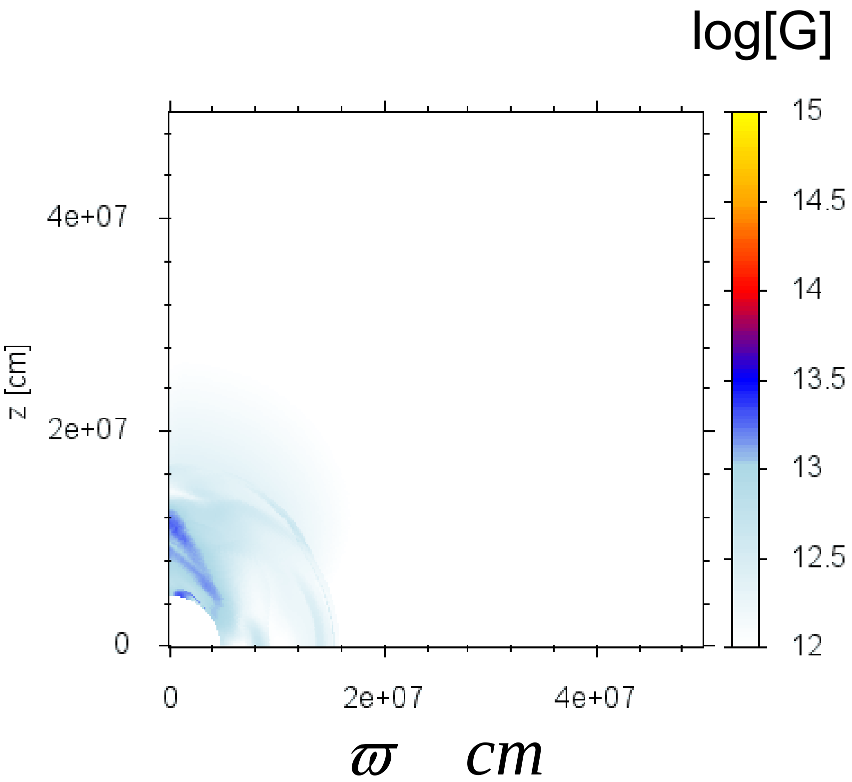


No coherent channel flows even during linear growth phase in our global simulation.

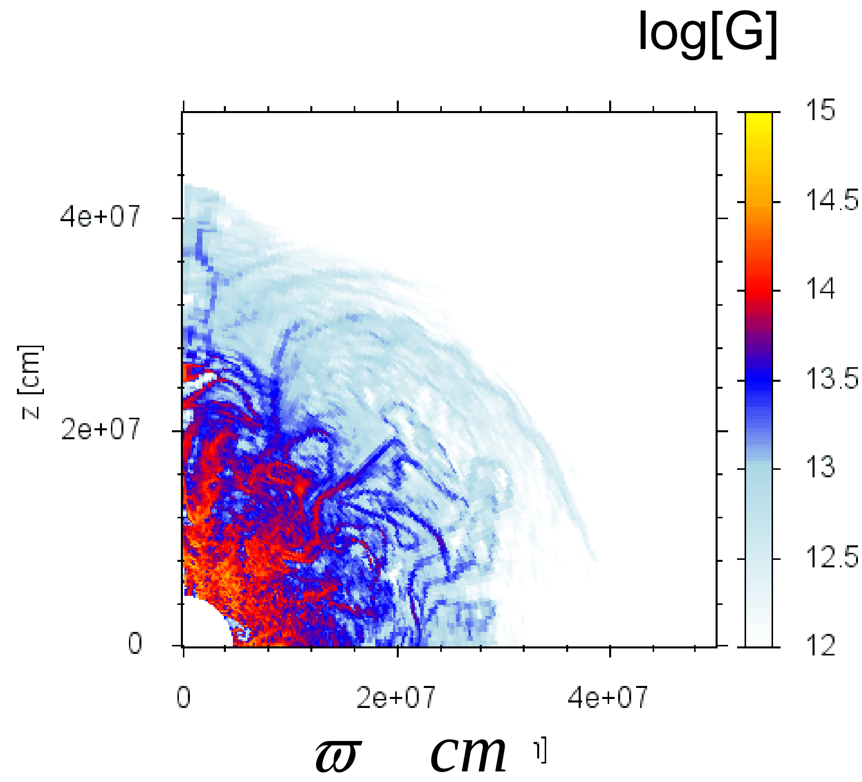
Δr_{min} = 13 m

Distributions of the strength
of the poloidal B-field

Beginning （1 ms）



After saturation（71 ms）



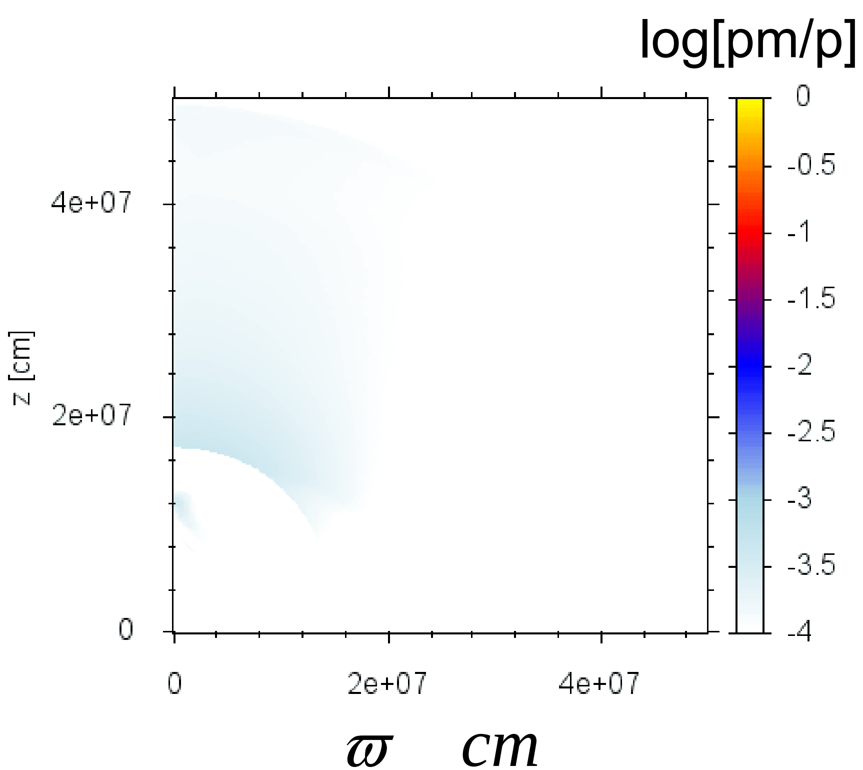
A magnetar-class magnetic field is generated.

◆Effect of B-field on the dynamics

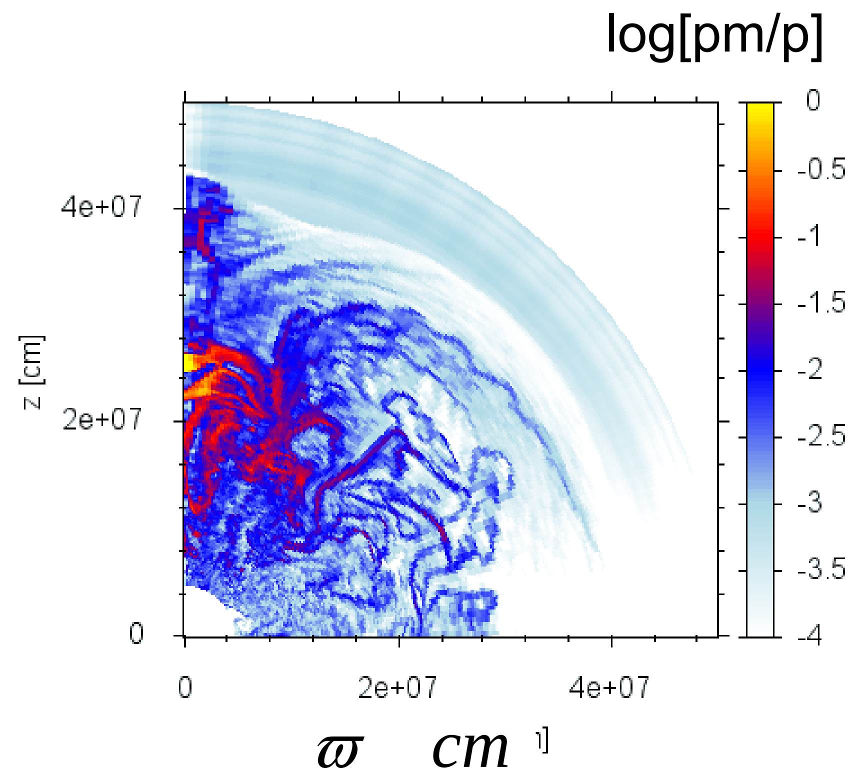
Δr_{min} = 13 m

Ratio of magnetic pressure to matter pressure

Beginning （1 ms）



After saturation（71 ms）



The amplified B-filed locally affects the dynamics.

4. Summary

We have performed the 2D-axisymmetric MHD simulations on a CCSN for a weakly magnetized, rapidly rotating progenitor.

- ✓ The first global simulation on MRI from sub-magnetar-class B-field.
- ✓ The B-field is exponentially amplified to magnetar-class strength.
- ✓ The amplified B-field locally affects the dynamics.
- ✓ No coherent channel flows appears as in local simulations.

Future works

- ✓ Parametric study varying
  initial B-field strength and the position of the inner boundary.
- ✓ Simulations on B-field amplification by convection.
- ✓ Non-axisymmetric simulations（3D-simulations）