Hall Equilibria

Solutions with toroidal and poloidal magnetic fields in Neutron Star Crusts

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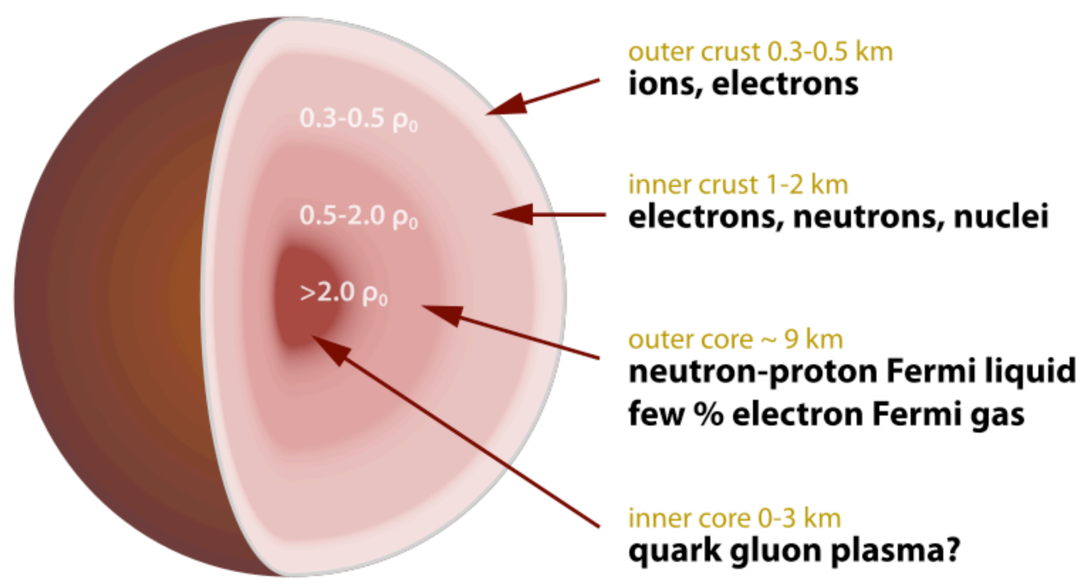
Abstract

We present equilibria solutions for neutron stars crusts containing toroidal and poloidal magnetic field. They correspond to Hall equilibria and are confined in the crust of the neutron star. Some simple cases are solved analytically while more complicated configurations are found numerically through a Gauss-Seidel elliptic partial differential equation solver.

1. Neutron Star Crusts

The crust is the upper layer of a neutron star (~2km) and consists of a highly conducting crystal lattice. Lorentz forces exerted by the magnetic field are balanced by the elastic crust and the field evolves through Hall effect[1].



2. Hall Evolution

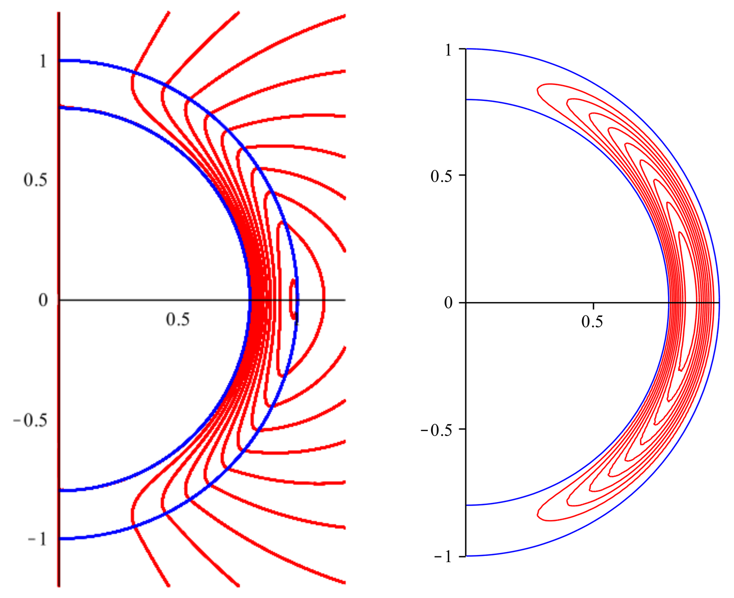
In the Hall description the magnetic field is advected by the electric current which is carried by electrons. Ohmic diffusion for neutron star crust applications is much slower and is treated as a secondary effect.

$$\frac{\partial \mathbf{B}}{\partial t} = \underbrace{-\frac{c}{4\pi e}\nabla\times\left(\frac{\nabla\times\mathbf{B}}{n_e}\times\mathbf{B}\right)}_{\text{Hall term}} \underbrace{-\frac{c^2}{4\pi}\nabla\times\left(\frac{1}{\sigma}\nabla\times\mathbf{B}\right)}_{\text{Ohmic term}}$$

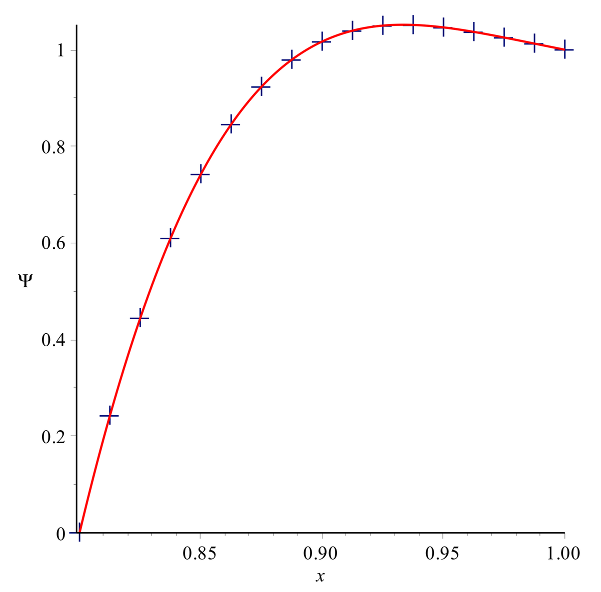
In a system dominated by Hall drift, equilibrium occurs when the Hall term is equal to zero and is the state it may settle after several Hall timescales. We solve for Hall equilibrium inside the neutron star crust while requiring that the field connects to an external vacuum dipole field.

3. Analytical Solutions

We find Hall equilibria solving the Grad-Shafranov[2] equation. The fields have an overall dipole structure and correspond to uniform rotation of the electron fluid. Analytical solutions are possible for purely poloidal field or for mixed poloidal-toroidal fields fully confined in the crust and bear similarities with previously known MHD solutions[3,4].



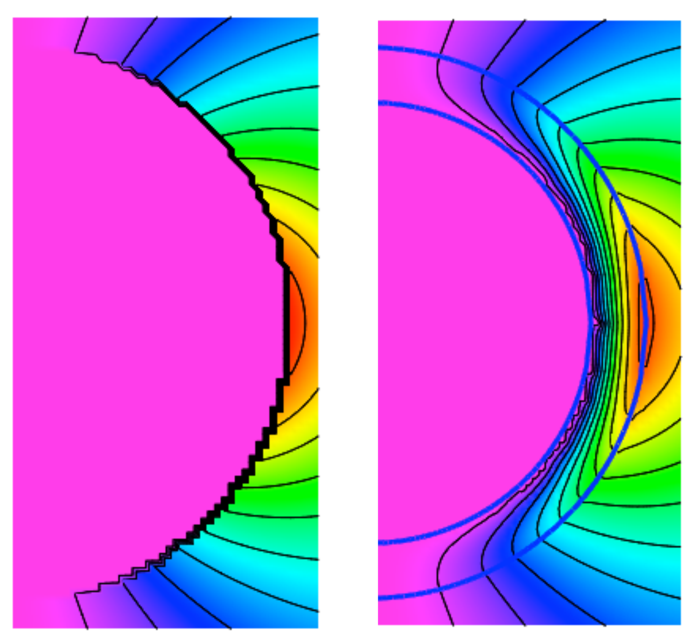
Analytical solutions for a pure dipole field and mixed toroidal and poloidal field fully confined inside the star.



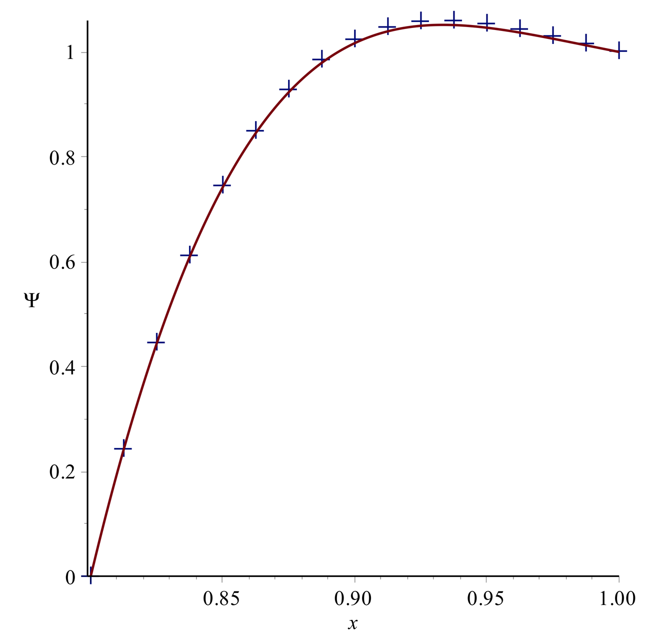
Comparison of the poloidal flux found analytically (red solid line) and the numerical solution (blue crosses).

4. Numerical Solutions

To avoid the essential simplifications of analytical solutions we numerically solve the Grad-Shafranov equation. The azimuthal field is constrained in closed tori as the external vacuum cannot support currents. The numerical scheme reproduces the analytical results and is in accordance with Hall simulations[5,6].



Left: Numerical solution initial condition. **Right**: The system relaxes to a Hall equilibrium where the toroidal field is confined in the closed tori.



Comparison of the purely poloidal solution with numerical solution with toroidal field.

5. Neutron Star Magnetic Evolution

The solutions presented, subject to their stability, may represent long term states in the magnetic evolution of neutron stars. Despite their similarities, they do not coincide with MHD equilibria, thus a phase transition during the formation of a neutron star from a fluid to a lattice state will be accompanied by magnetic activity. For long enough timescales Ohmic diffusion may become important and the neutron star shall deviate from Hall equilibrium.

References

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