Fast Radio Bursts in the Fireball Paradigm

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Nishiura, Kamijima, Iwamoto & KI 24

KI 20, Wada & KI 23, Ishizaki & KI 24

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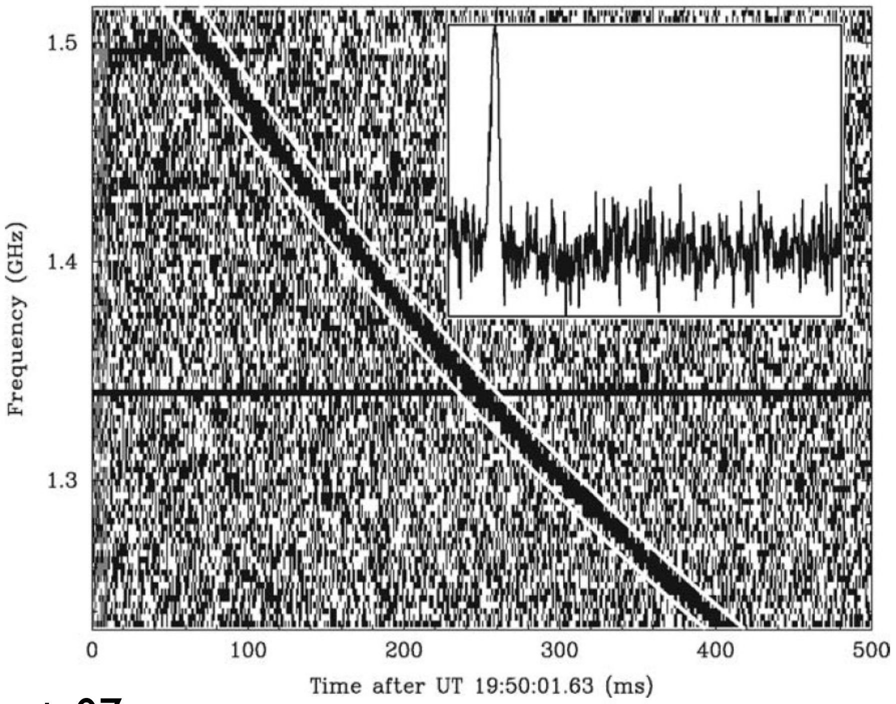
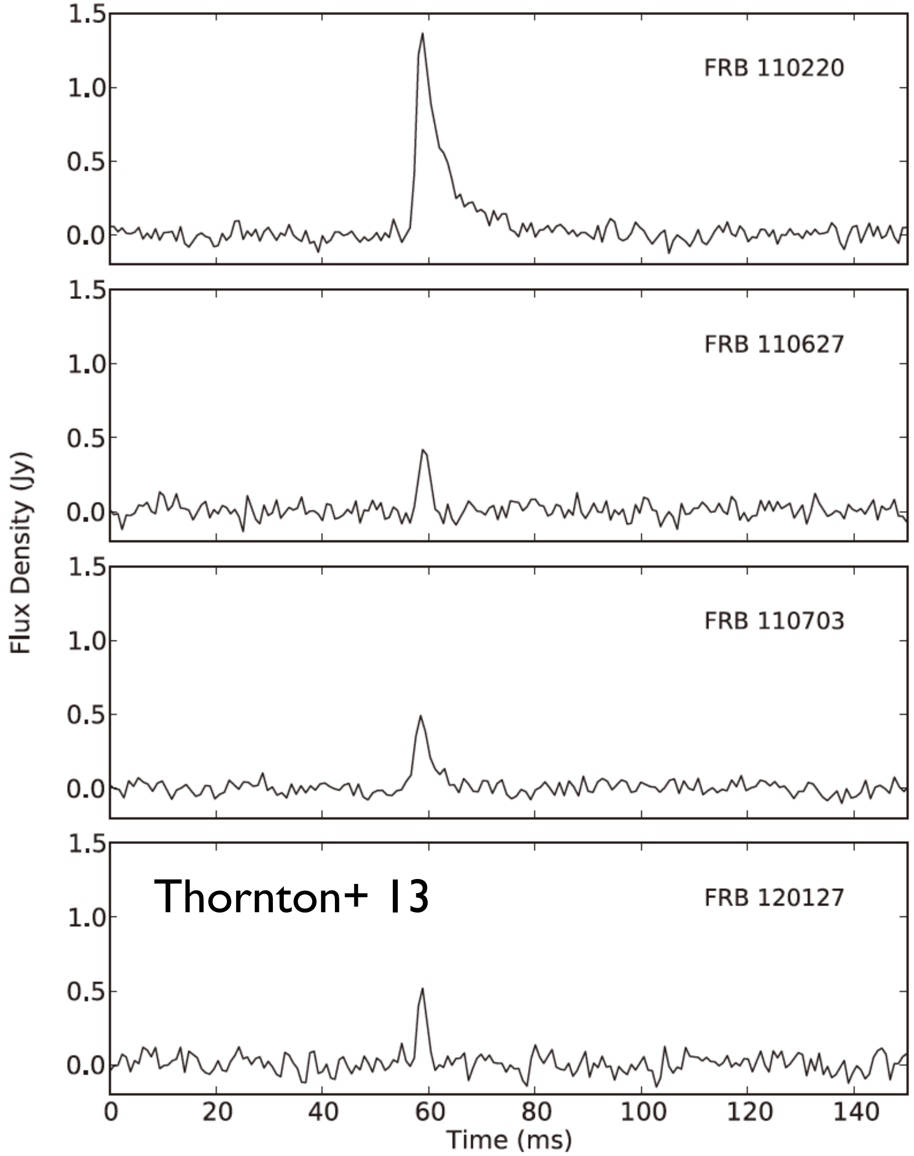
Fast Radio Bursts (FRB)

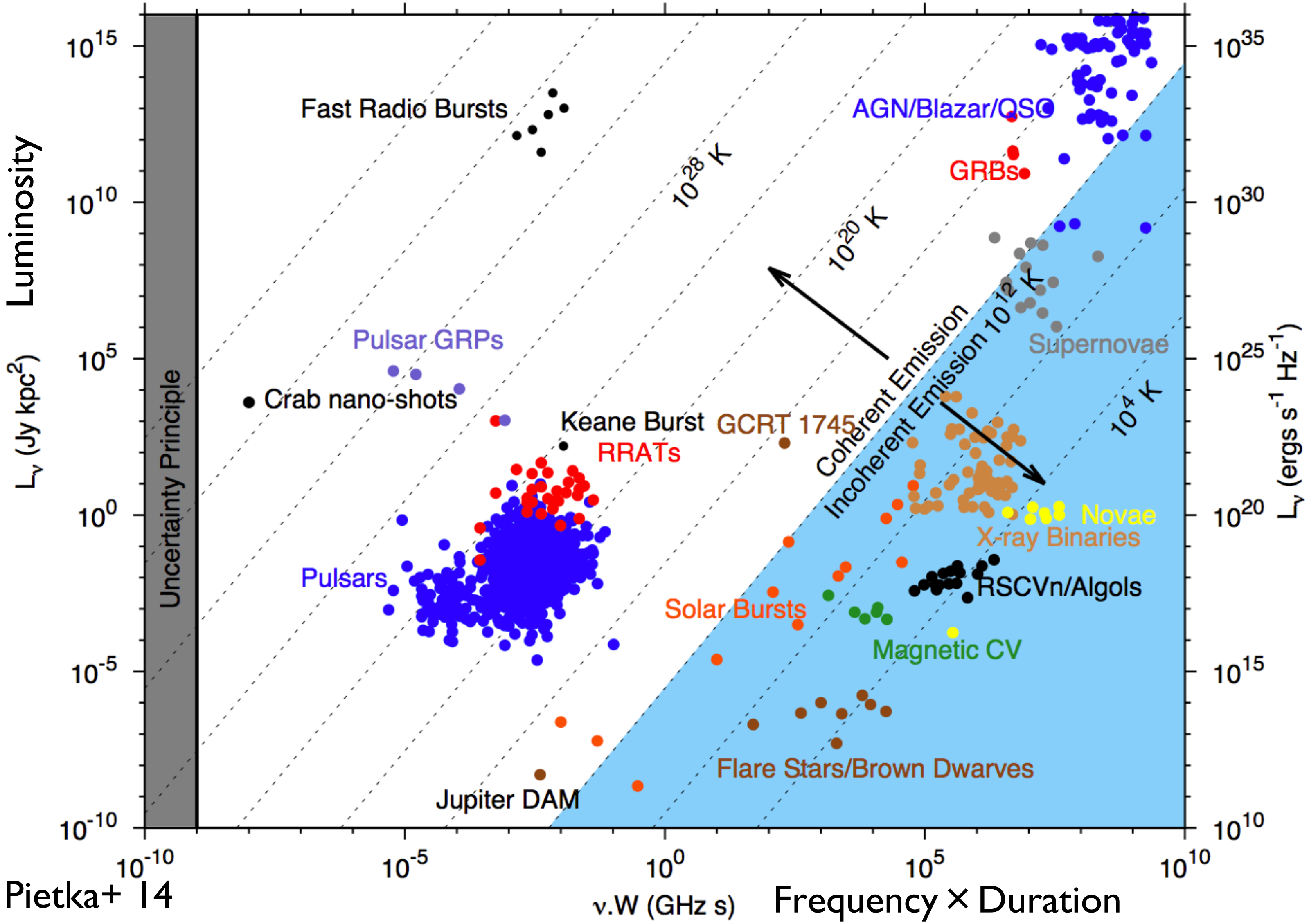


Lorimer+ 07



Most luminous radio transients discovered in 2007

Brightness Temperature



Pietka+ 14

Brightness temperature

$$T = \frac{c^2 I_\nu}{2k\nu^2} > 10^{35}\mathrm{K}\frac{F_{\nu,\mathrm{Jy}} d_{\mathrm{Gpc}}^2}{\Delta t_{\mathrm{ms}}^2 \nu_{\mathrm{GHz}}^2}$$

$$F_\nu \simeq I_\nu \Delta\Omega \simeq I_\nu \frac{\pi \ell^2}{d^2}$$

$$\ell < c\Delta t$$

$$\to \nu = \Gamma\nu', I_\nu/\nu^3 = I'_{\nu'}/\nu'^3, \ell < c\Gamma\Delta t$$

$$T' > 10^{35}\mathrm{K}\frac{F_{\nu,\mathrm{Jy}} d_{\mathrm{Gpc}}^2}{\Delta t_{\mathrm{ms}}^2 \nu_{\mathrm{GHz}}^2 \Gamma^3}$$

Coherent number

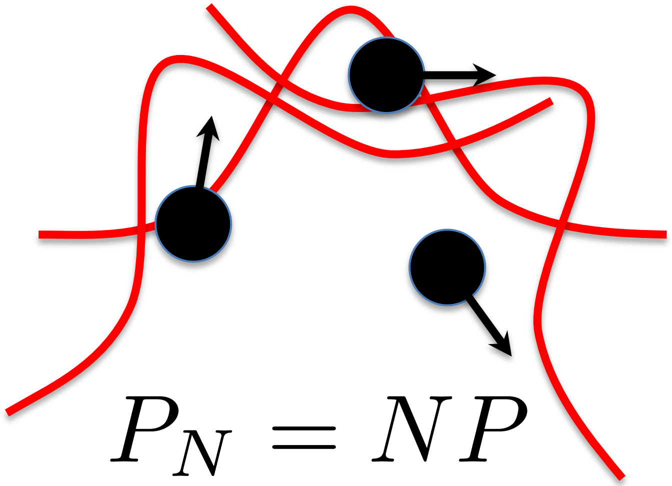
$$\mathcal{N} \sim \frac{kT'}{\gamma m_e c^2} \sim 10^{25}\frac{F_{\nu,\mathrm{Jy}} d_{\mathrm{Gpc}}^2}{\Delta t_{\mathrm{ms}}^2 \nu_{\mathrm{GHz}}^2 \gamma \Gamma^3}$$

Coherent Emission

$$P_N = \left| \sum_{k=1}^{N} E_k e^{i\phi_k} \right|^2$$

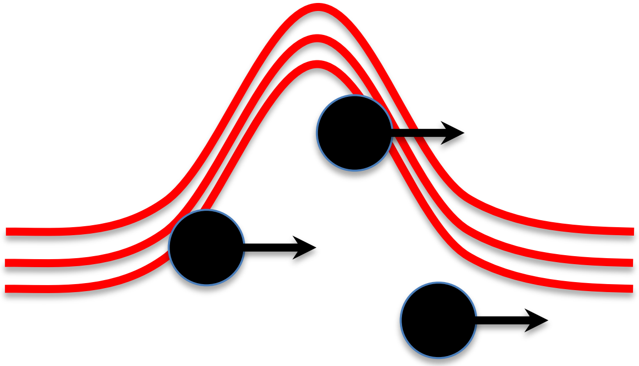
$$= N\,|E|^2 + |E|^2 \sum_{k \neq j} e^{i(\phi_k - \phi_j)}$$

Incoherent



$$P_N = NP$$

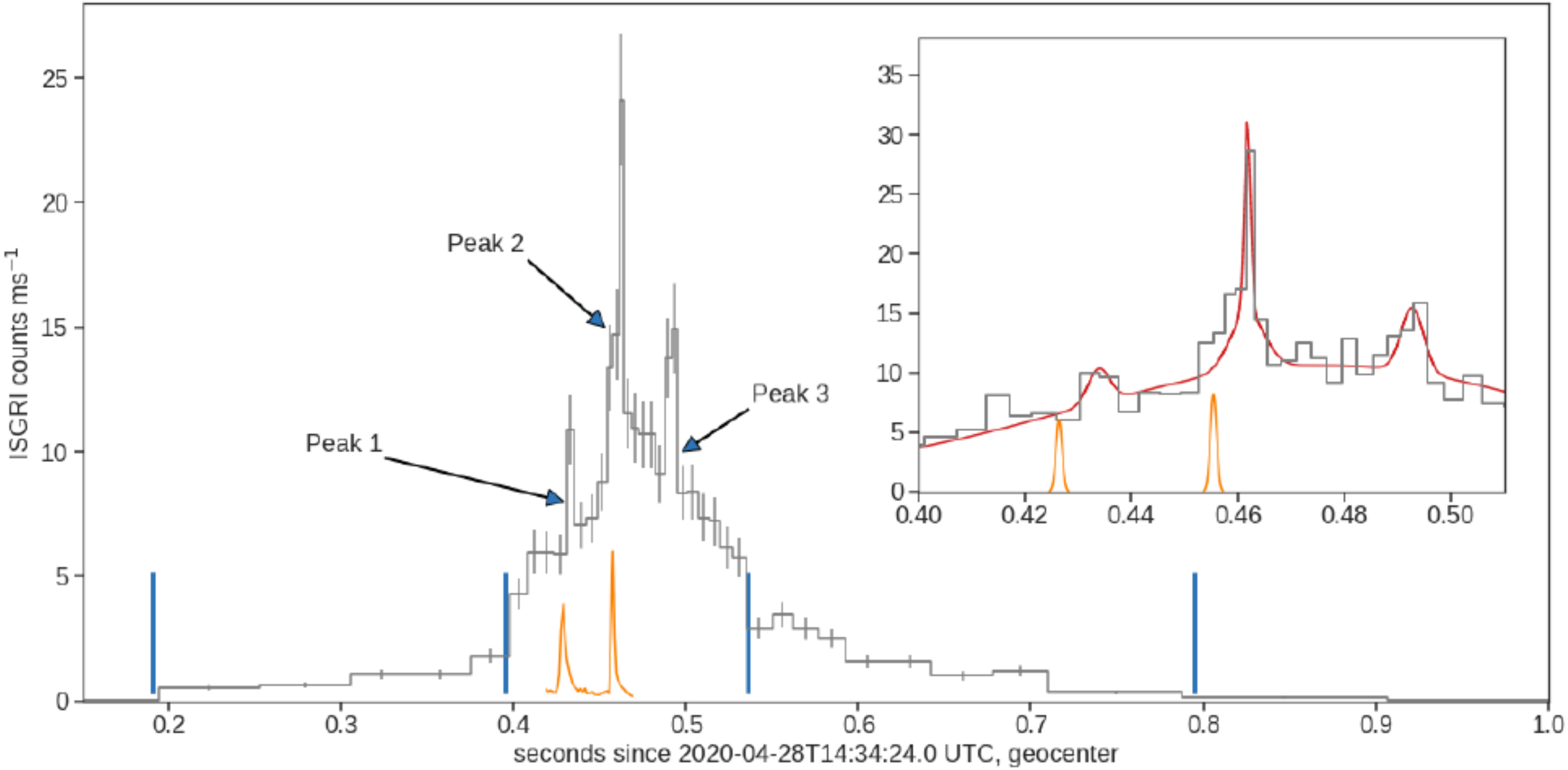
Coherent



$$P_N = N^2 P$$

Galactic FRB from Magnetar Bursts

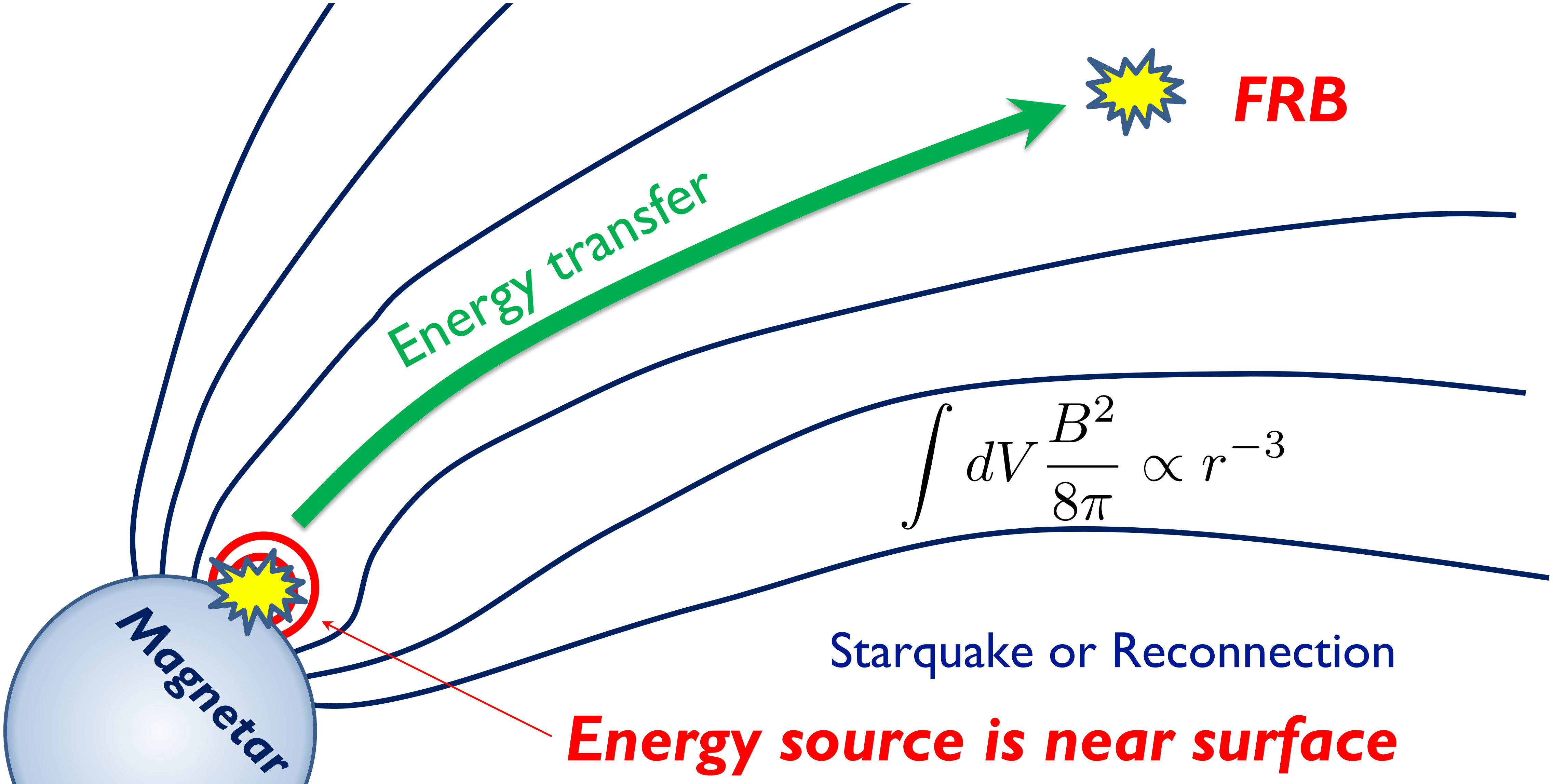
A smoking gun! Magnetar: One of the origins



$L_X \sim 10^{41}$ erg/s >> $L_{FRB} \sim 10^{38}$ erg/s

Mereghetti+ 20,
Bochenek+ 20,
CHIME/FRB+ 20,
Li+ 20,
Ridnaia+ 20,
Tavani+ 20

Kinetic? Magnetic?

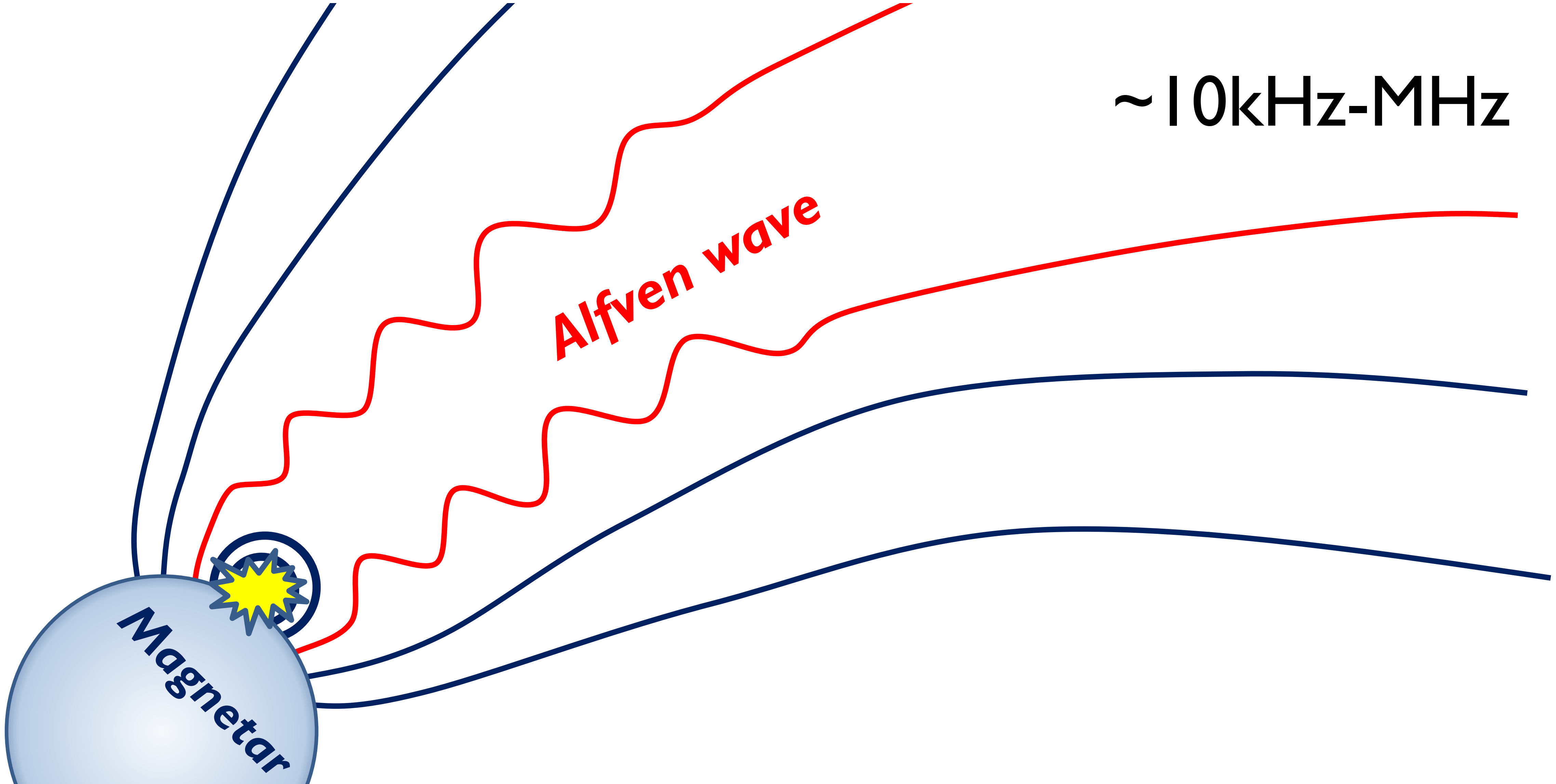


$$\int dV \frac{B^2}{8\pi} \propto r^{-3}$$

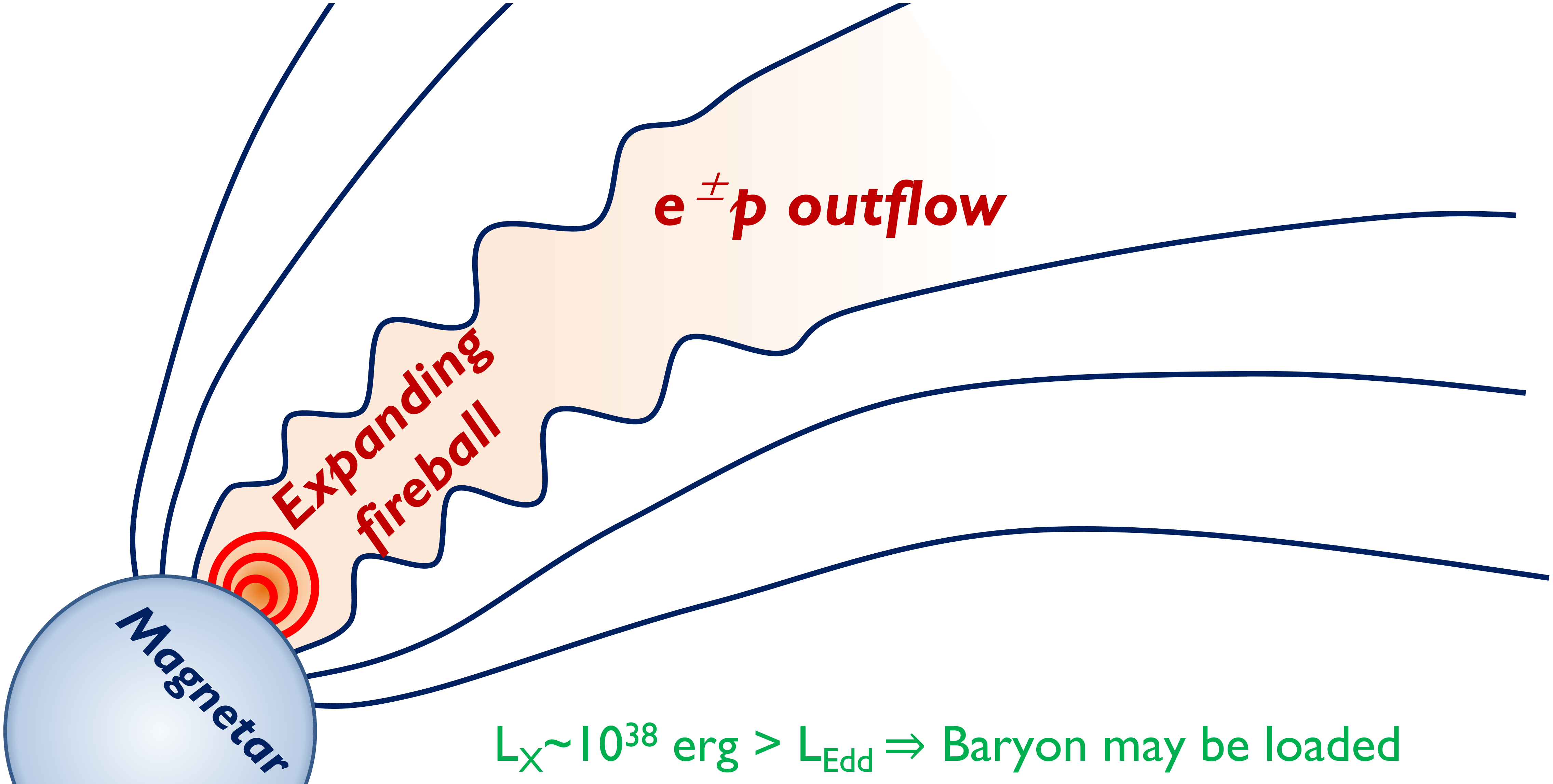
Starquake or Reconnection

Energy source is near surface

Magnetic Pulse: Poynting Flux

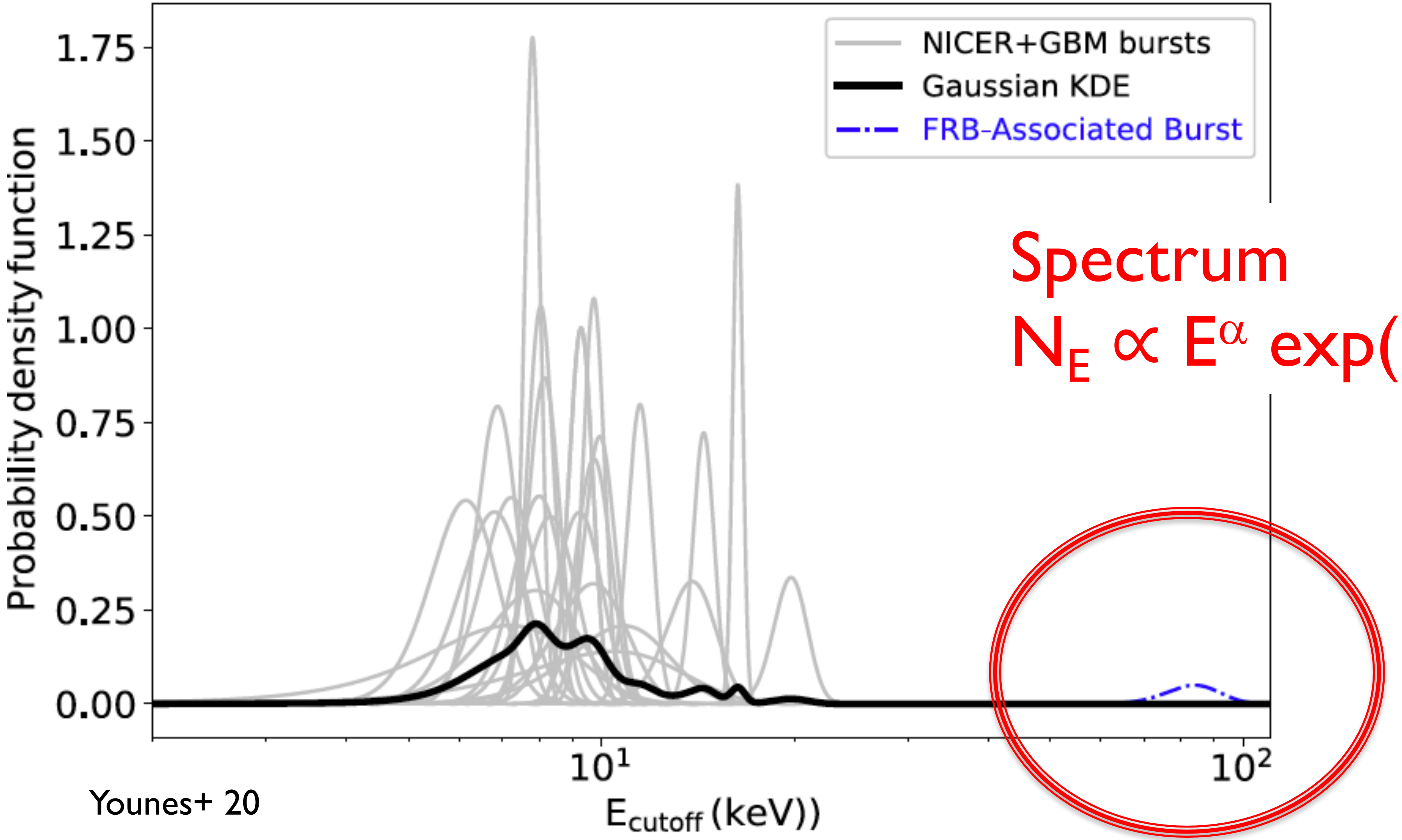


Fireball: Kinetic Flux



$L_X \sim 10^{38}$ erg > L_{Edd} ⇒ Baryon may be loaded

High Temperature

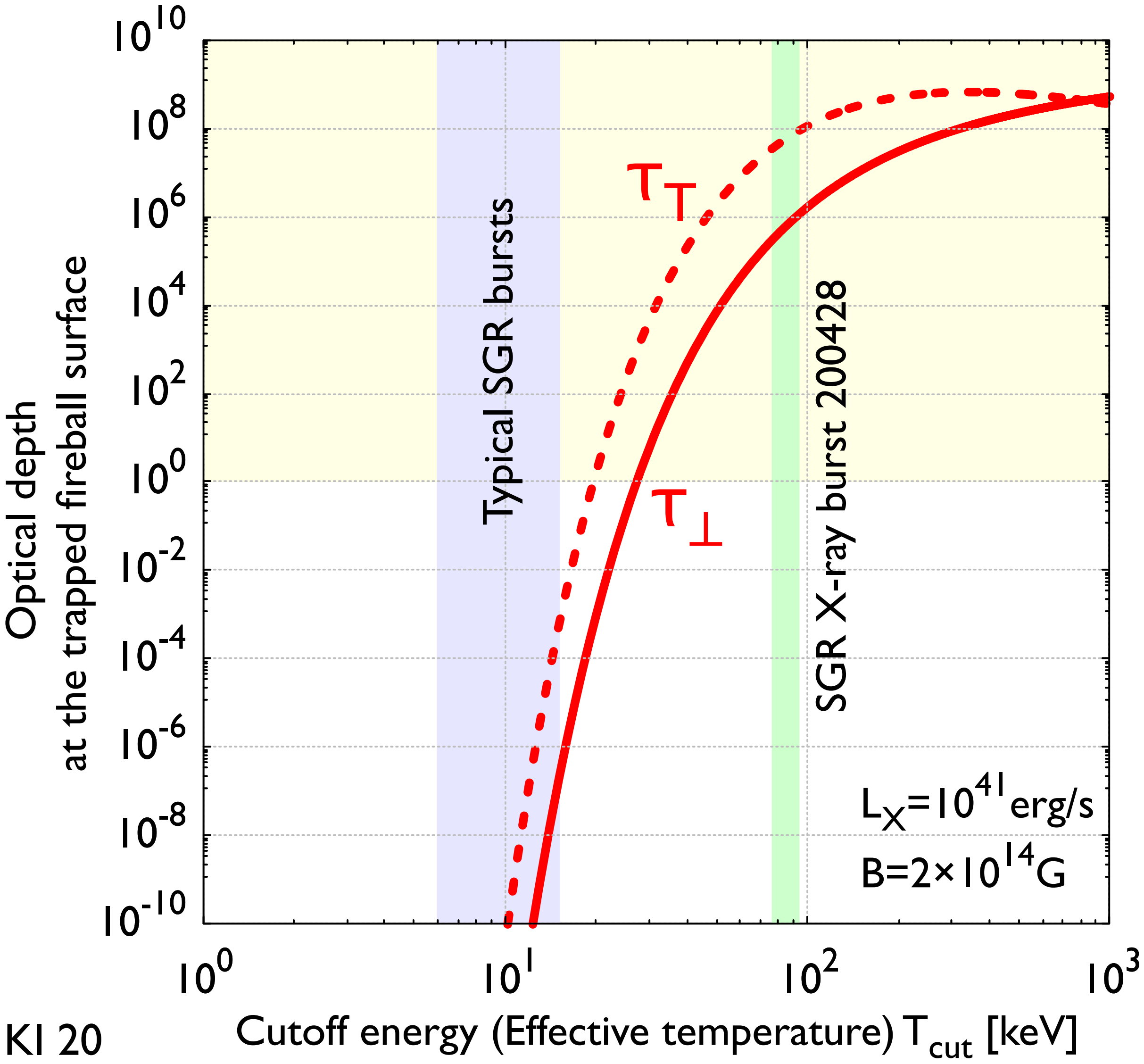


Spectrum

$N_E \propto E^\alpha \exp(-E/E_{cut})$

Younes+ 20

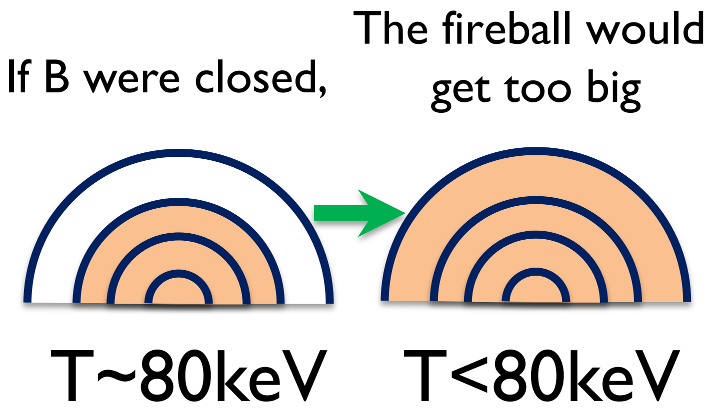
Optical Depth



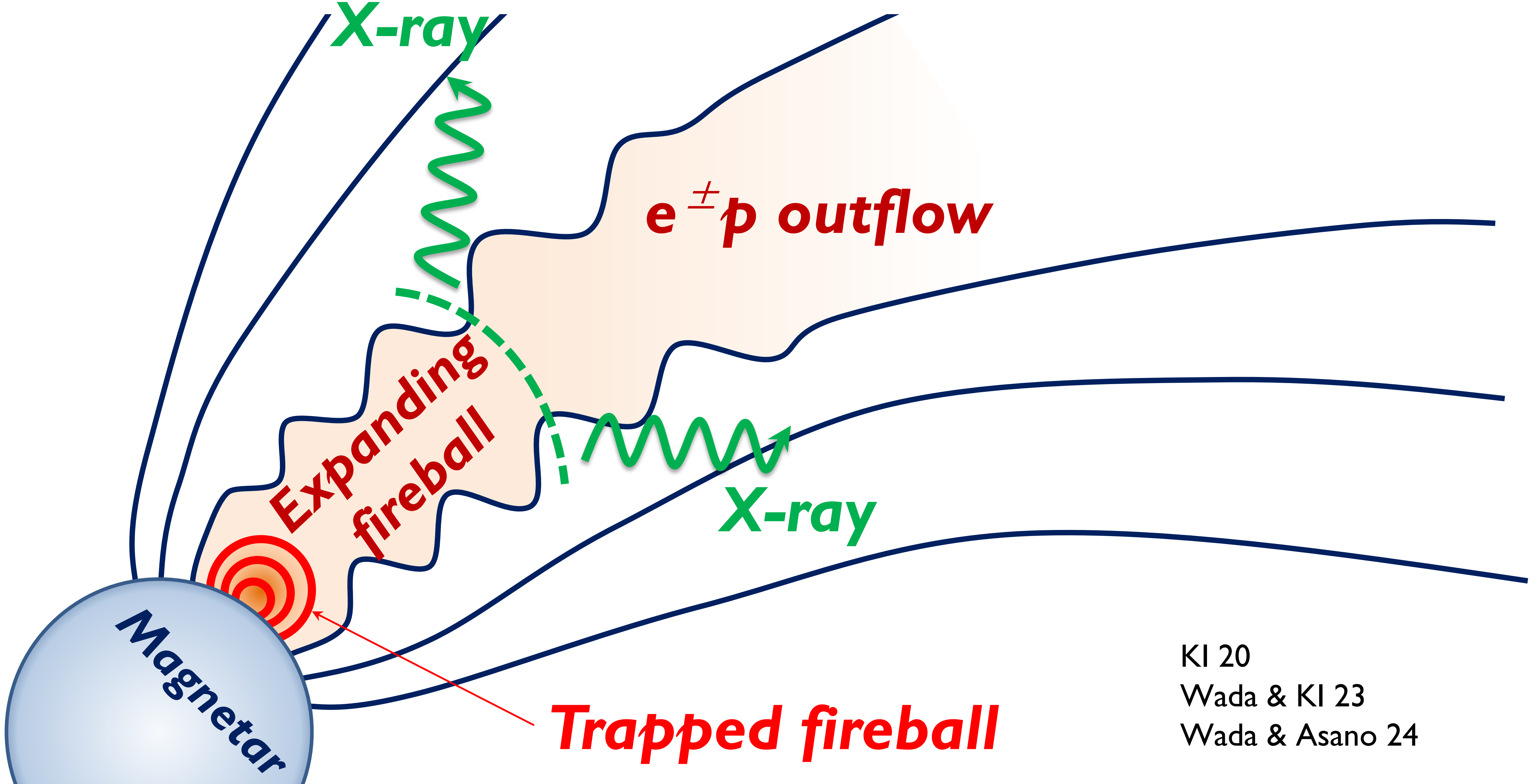
KI 20

τ **>>1** at the surface of the trapped fireball

X-ray tails create $e^{\pm}$
→ Surrounding field should be open
→ ***Expanding fireball***



Expanding Fireball



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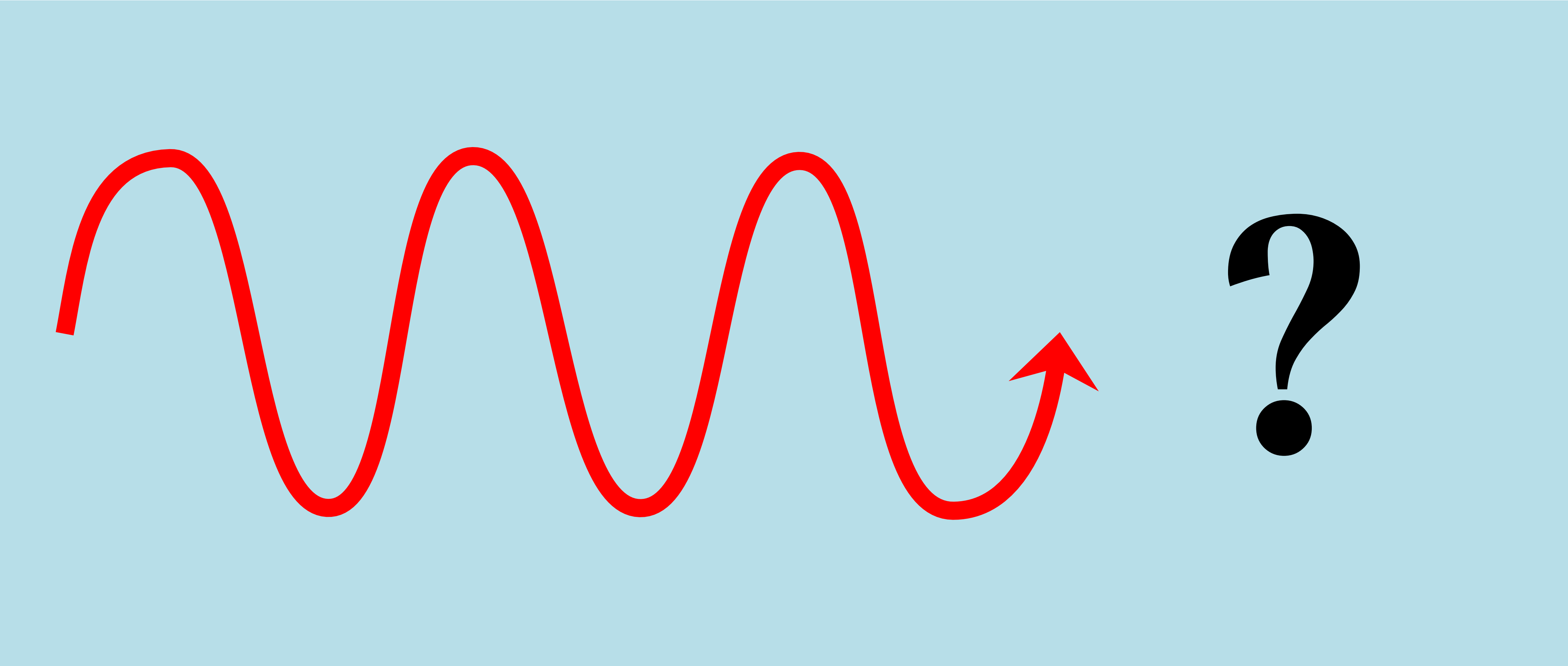
Wada & Asano 24

Wave in Plasma

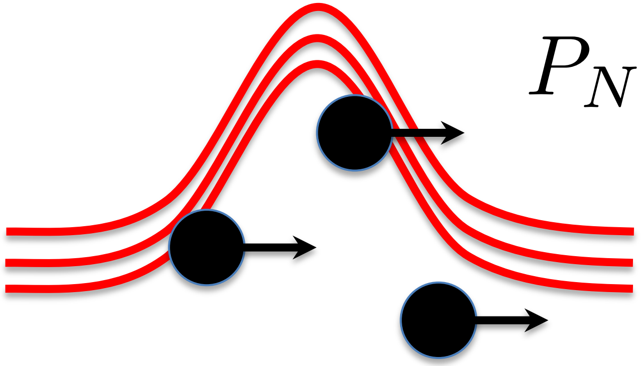


$\omega > \omega_p$ plasma frequency

Antenna vs. Maser

Antenna mechanism

$$f(\boldsymbol{x},\boldsymbol{p},t) \propto \delta(\boldsymbol{x})\delta(\boldsymbol{p})$$



$$P_N = N^2 P$$

N particles

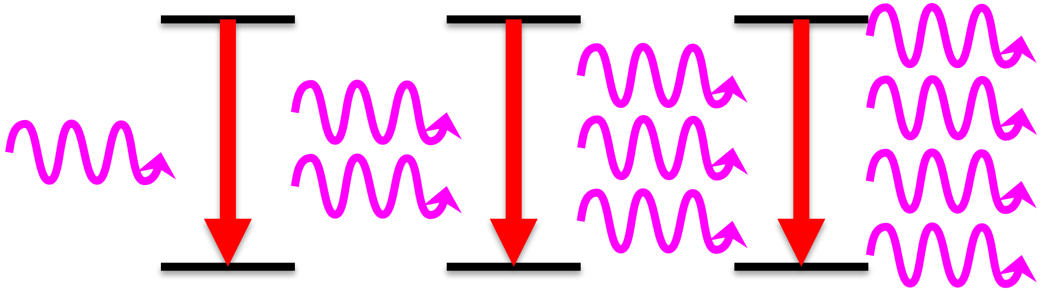
wavelength λ

Spontaneous

Maser mechanism ✔

$$f(\boldsymbol{x},\boldsymbol{p},t) \propto \delta(\boldsymbol{p})$$

or even w/o bunch

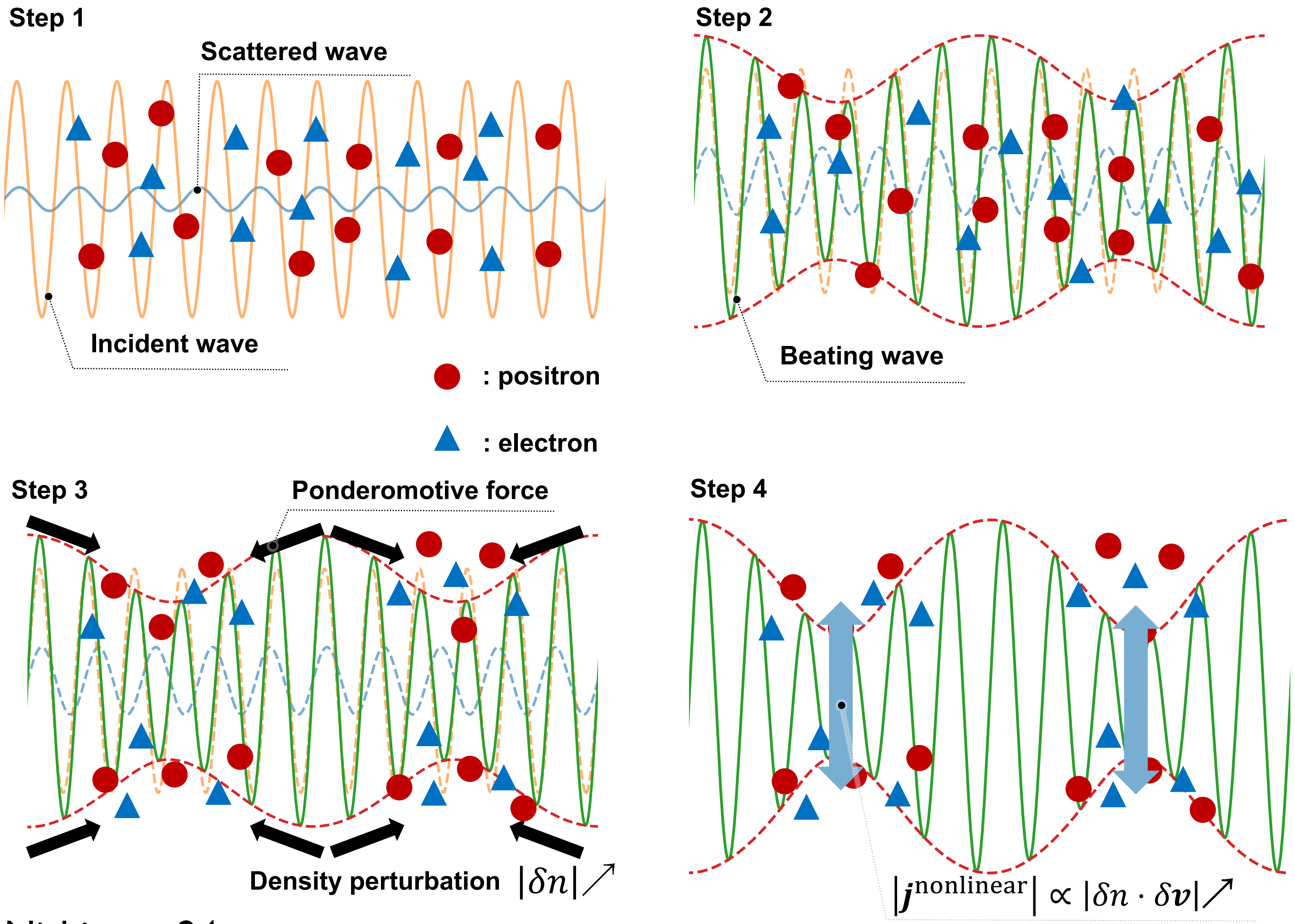


N particles

$>\lambda$

Induced (Stimulated)

Induced Scattering



Nishiura+ 24

Maser (induced emission)

Classical plasma process

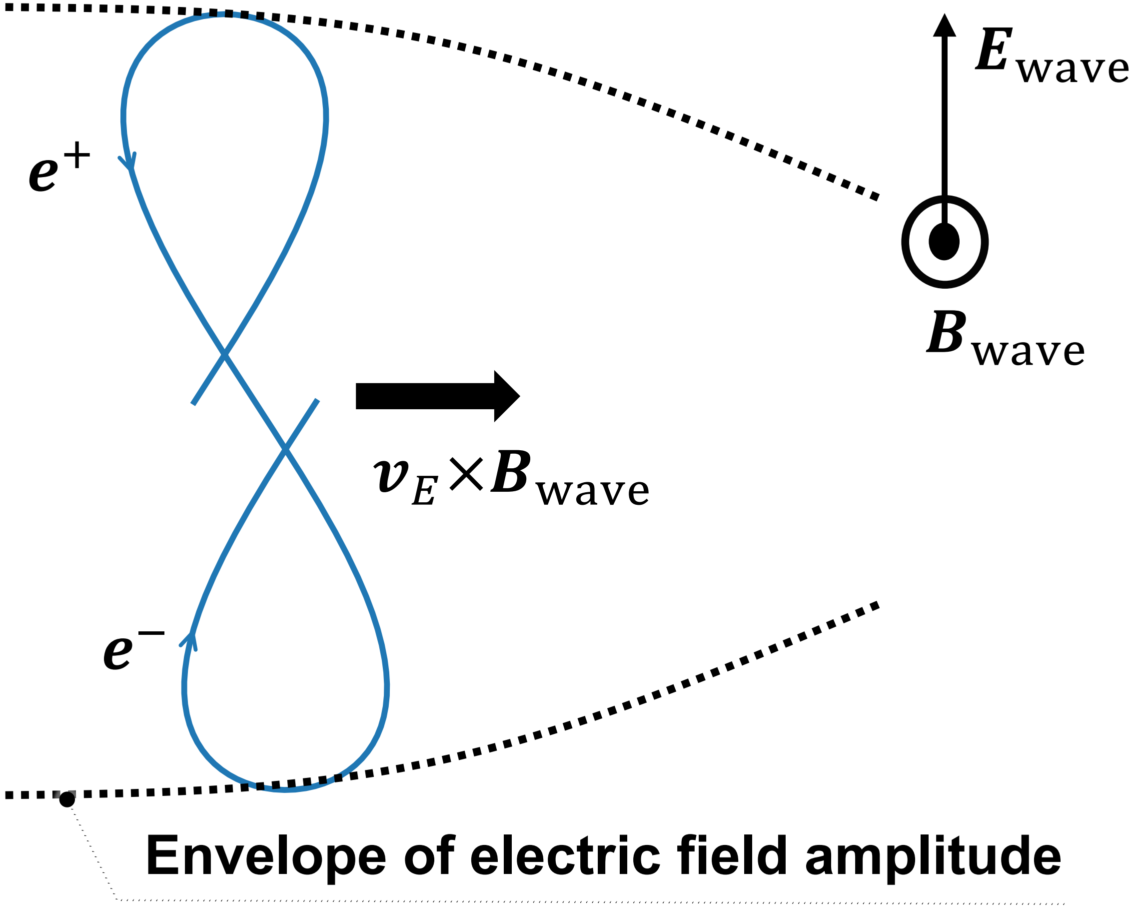
Parametric instability

- ***Induced Compton***
- Induced Brillouin (Ishizaki & KI 24)
- Induced Raman
- Filamentation instability

3 waves $\omega_0 = \omega_1 + |\omega|$

EM → EM + Density wave

Ponderomotive Force



$$\frac{d^2 \boldsymbol{r}}{dt^2} = q\boldsymbol{E}(\boldsymbol{r}) + \frac{q}{c}\boldsymbol{v} \times \boldsymbol{B}$$

$$\boldsymbol{r} = \underbrace{\boldsymbol{r}_0}_{\text{oscillation center}} + \underbrace{\boldsymbol{r}_1}_{\text{fast oscillation}}$$

$$\frac{d^2 \boldsymbol{r}_0}{dt^2} \simeq -\nabla \phi_{\rm p}$$

ponderomotive potential

$$\phi_{\rm p} = \frac{e^2}{2m\omega^2}\langle|\boldsymbol{E}(\boldsymbol{r}_0)|^2\rangle_{\rm time}$$

Independent of the charge sign

Growth Rate

$$\Gamma_{\rm C}^{\rm max} = \sqrt{\frac{\pi}{32e}\frac{\omega_{\rm p}^2 a_{\rm e}^2}{\omega_0}\frac{m_{\rm e}c^2}{k_{\rm B}T_{\rm e}}},$$

Strength parameter (dimensionless amplitude) $a_{\rm e} \equiv \frac{2e\,|\boldsymbol{A}_0|}{m_{\rm e}c^2},$

Plasma frequency $\omega_{\rm p} \equiv \sqrt{\frac{8\pi e^2 n_{\rm e0}}{m_{\rm e}}},$

Frequency of the most growing waves

$$\omega_1(\nu,\theta_{kB}) \simeq \omega_0\left(1-\sqrt{2(1-\nu)\cos^2\theta_{kB}\frac{k_{\rm B}T_{\rm e}}{m_{\rm e}c^2}}\right)$$ (for $\mathbf{A}_0 \| \mathbf{B}_0$)

Scattering rate

$$\left(t_{\rm c,\|}^{\rm broad}\right)^{-1} = \pi\frac{\omega_{\rm p}^2 a_{\rm e}^2}{\omega_0}\left(\frac{\omega_0}{\Delta\omega}\right)^2$$
$$= 1.1\times 10^{20}\,{\rm s}^{-1}\frac{\mathcal{M}_6 R_6^3 B_{\rm p,14} L_{38}}{P_{\rm sec} r_8^5 \nu_9^2}\left(\frac{\Delta\nu/\nu_0}{1}\right)^{-2} \gg \Delta t^{-1}$$

Inverse of the burst duration

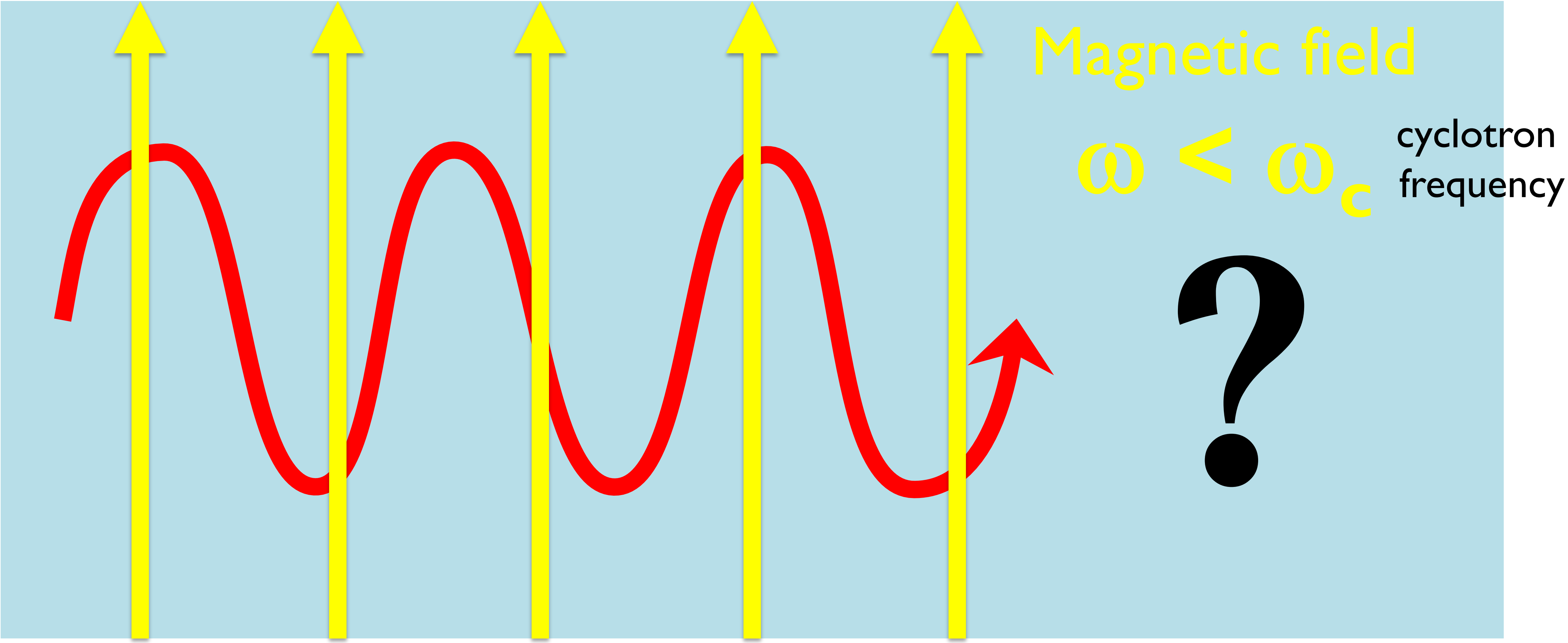
$$\Delta t^{-1} = 10^3\ {\rm s}^{-1}$$

➡ Many scatterings

➡ Dissipation

Lyubarsky 08
Ghosh+ 21

Wave in Plasma



$\omega < \omega_p$ (plasma frequency) can propagate

Basic Equations

Maxwell eq.

$$\frac{\partial^2 \boldsymbol{A}}{\partial t^2} - c^2 \Delta \boldsymbol{A} = 4\pi c \boldsymbol{j}$$

Equations of motion ($\omega \sim \omega_{0,1}$)

$$\frac{d\boldsymbol{v}_\pm}{dt} = \pm \frac{e}{m_e}\left(\boldsymbol{E} + \frac{\boldsymbol{v}_\pm \times \boldsymbol{B}_0}{c}\right)$$

Vlasov equation ($\omega \ll \omega_{0,1}$)

$$\frac{\partial f_\pm}{\partial t} + \boldsymbol{v}\cdot\nabla f_\pm + \boldsymbol{F}\cdot\frac{\partial f_\pm}{\partial \boldsymbol{p}} = 0$$

$$\boldsymbol{F} = -\nabla\phi_\pm \pm e\left(\boldsymbol{E} + \frac{\boldsymbol{v}\times\boldsymbol{B}_0}{c}\right)$$

Ponderomotive force

$$\nabla\cdot\boldsymbol{E} = \sum_{q=\pm e} 4\pi q n_{e0}\int \delta f_\pm d^3\boldsymbol{v}$$

EM waves $\boldsymbol{A}(\boldsymbol{r},t) = \boldsymbol{A}_0 e^{i(\boldsymbol{k}_0\cdot\boldsymbol{r}-\omega_0 t)} + \boldsymbol{A}_1 e^{i(\boldsymbol{k}_1\cdot\boldsymbol{r}-\omega_1 t)} + \text{c.c.},$

Density fluctuation $\widetilde{\delta n_\pm}(\boldsymbol{k},\omega)$

$$= n_{e0}\int d^3\boldsymbol{v}\,\widetilde{\delta f_\pm}(\boldsymbol{k},\boldsymbol{v},\omega)$$

Current

$$\boldsymbol{j} = \sum_{q=\pm e} q n_\pm(\boldsymbol{r},t)\boldsymbol{v}_\pm(\boldsymbol{r},t)$$

➡ Dispersion relation for (ω_1, k_1)

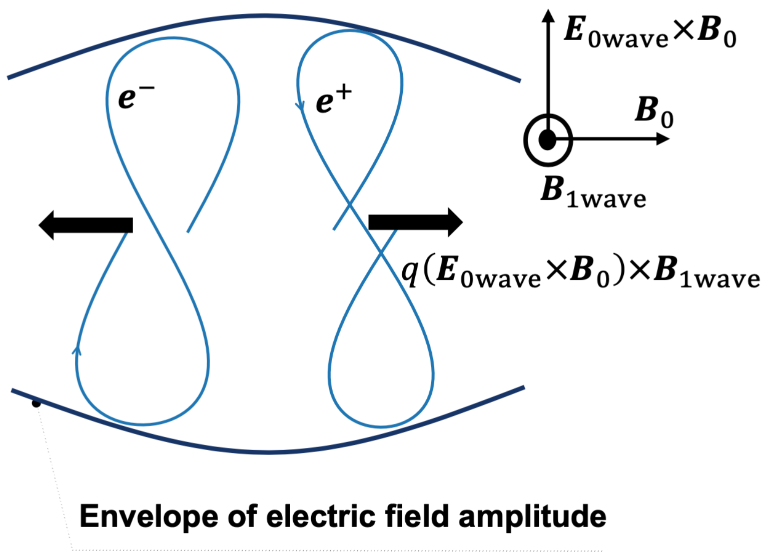
(for $\mathbf{A}_0 \| \mathbf{B}_0$)

$$c^2 k_1^2 - \omega_1^2 + \omega_p^2 = \frac{1}{4}c^2(\omega_p a_e \mu)^2 \times \sum_{\ell=-\infty}^{+\infty}\int d^3\boldsymbol{v}\frac{J_\ell^2(k_\perp r_L)\,\boldsymbol{k}\cdot\frac{\partial f_{0\pm}}{\partial \boldsymbol{v}^*}}{\omega - k_\parallel v_\parallel - \ell\omega_c}.$$

Growth Rate

Charged mode

Drift

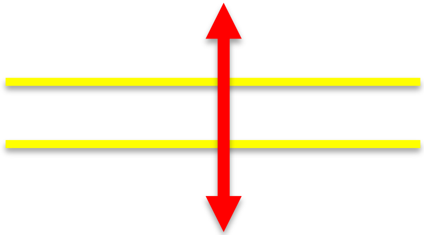


Induced Compton scattering for $A_{0\perp} = 0$ (narrow band)

① Ordinary mode

$$\Gamma_{\mathrm{C}}^{\max} = \sqrt{\frac{\pi}{32\mathrm{e}}} \frac{\omega_{\mathrm{p}}^2 a_{\mathrm{e}}^2}{\omega_0} \frac{m_{\mathrm{e}} c^2}{k_{\mathrm{B}} T_{\mathrm{e}}}$$

Induced Compton scattering for $A_{0\parallel} = 0$ (narrow band)

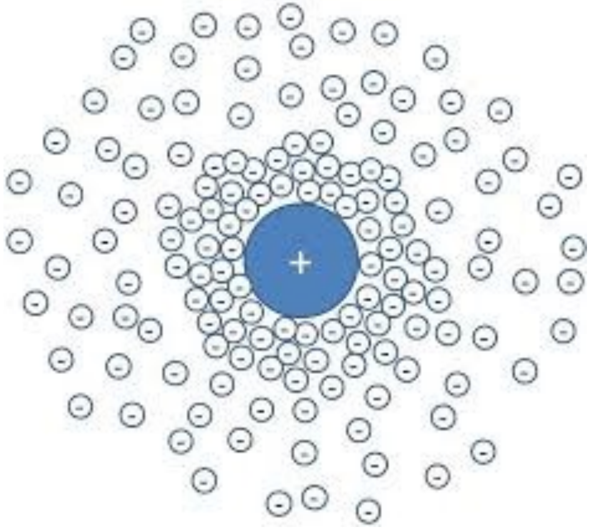


② Charged mode

$$\Gamma_{\mathrm{C}}^{\max} = \sqrt{\frac{\pi}{32\mathrm{e}}} \left(\frac{\omega_0}{\omega_{\mathrm{c}}}\right)^2 \frac{\omega_{\mathrm{p}}^2 a_{\mathrm{e}}^2}{\omega_0} \frac{m_{\mathrm{e}} c^2}{k_{\mathrm{B}} T_{\mathrm{e}}} \times \begin{cases} 1 & \frac{8 k_{\mathrm{B}} T_{\mathrm{e}}}{m_{\mathrm{e}} c^2} \left(\frac{\omega_0}{\omega_{\mathrm{p}}}\right)^2 \geq 1 \\ \frac{\mathrm{e}}{2\pi} \left(\frac{\omega_0}{\omega_{\mathrm{p}}}\right)^4 \left(\frac{8 k_{\mathrm{B}} T_{\mathrm{e}}}{m_{\mathrm{e}} c^2}\right)^2 & \frac{8 k_{\mathrm{B}} T_{\mathrm{e}}}{m_{\mathrm{e}} c^2} \left(\frac{\omega_0}{\omega_{\mathrm{p}}}\right)^2 \ll 1 \end{cases}$$

Gyroradius effect

Debye screening effect



③ Neutral mode

$$\Gamma_{\mathrm{C}}^{\max} = \sqrt{\frac{\pi}{32\mathrm{e}}} \left(\frac{\omega_0}{\omega_{\mathrm{c}}}\right)^4 \frac{\omega_{\mathrm{p}}^2 a_{\mathrm{e}}^2}{\omega_0} \frac{m_{\mathrm{e}} c^2}{k_{\mathrm{B}} T_{\mathrm{e}}}$$

(Gyroradius effect)^2

$$\frac{8 k_{\mathrm{B}} T_{\mathrm{e}}}{m_{\mathrm{e}} c^2} \left(\frac{\omega_0}{\omega_{\mathrm{p}}}\right)^2 = 32\pi^2 \left(\frac{\lambda_{\mathrm{De}}}{\lambda_0}\right)^2$$

Scattering Rate

Scattering rate

$$\left(t_{\rm c,\parallel}^{\rm broad}\right)^{-1} = \pi \frac{\omega_{\rm p}^2 a_{\rm e}^2}{\omega_0}\left(\frac{\omega_0}{\Delta\omega}\right)^2$$
$$= 1.1\times 10^{20}\,{\rm s}^{-1}\frac{\mathcal{M}_6 R_6^3 B_{\rm p,14} L_{38}}{P_{\rm sec} r_8^5 \nu_9^2}\left(\frac{\Delta\nu/\nu_0}{1}\right)^{-2} \gg \Delta t^{-1}$$

Inverse of the burst duration

$$\Delta t^{-1} = 10^3\ {\rm s}^{-1}$$

→
$$\left(t_{\rm charged}^{\rm broad}\right)^{-1} = 32\pi\left(\frac{\omega_0}{\omega_{\rm c}}\right)^2 \frac{\omega_{\rm p}^2 a_{\rm e}^2}{\omega_0}\left(\frac{k_{\rm B}T_{\rm e}}{m_{\rm e}c^2}\right)^2\left(\frac{\omega_0}{\omega_{\rm p}}\right)^4\left(\frac{\omega_0}{\Delta\omega}\right)^2$$
$$= 9.3\times 10^2\,{\rm s}^{-1}\frac{P_{\rm sec} r_8^7 L_{38} T_{\rm 80keV}^2}{\mathcal{M}_6 \nu_9^2 R_6^9 B_{\rm p,14}^3}\left(\frac{\Delta\nu/\nu_0}{1}\right)^{-2} \sim \Delta t^{-1},$$

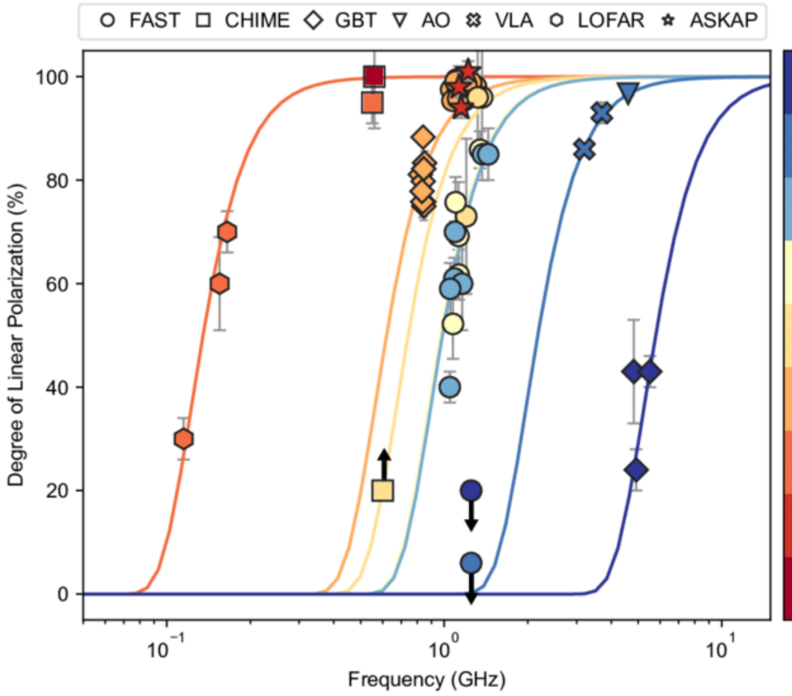
$$t_{\rm C,wrong}^{-1} \sim \frac{(\omega_{\rm p} a_{\rm e})^2}{\omega_0}\left(\frac{\omega_0}{\omega_{\rm c}}\right)^2$$
$$\sim 4.5\times 10^8\,{\rm s}^{-1}\,\frac{r_8\,L_{38}\,\mathcal{M}_6\,R_{\rm NS,6}^3}{B_{\rm p,14}\,\nu_9\,P_{\rm sec}}$$

→
$$\left(t_{\rm neutral}^{\rm broad}\right)^{-1} = \pi \frac{\omega_{\rm p}^2 a_{\rm e}^2}{\omega_0}\left(\frac{\omega_0}{\omega_{\rm c}}\right)^4\left(\frac{\omega_0}{\Delta\omega}\right)^2$$
$$= 1.8\times 10^{-2}\,{\rm s}^{-1}\frac{\mathcal{M}_6 L_{38}\nu_9^2 r_8^7}{P_{\rm sec} R_6^9 B_{\rm p,14}^3}\left(\frac{\Delta\nu/\nu_0}{1}\right)^{-2} \ll \Delta t^{-1}$$

Waves can escape!

Polarization

	Scatt. angle	Escaping polarization	Max
Ordinary mode	$E_1 \parallel E_0$	$E \perp B_0$	***100%***
Charged mode	$E_1 \perp E_0$	$E \perp B_0$	***~50%***
Neutral mode	$E_1 \parallel E_0$	$E \perp B_0$	***~50%***



Feng+ 22
Gajjar+ 18
Michilli+ 18
Oslowski+ 18

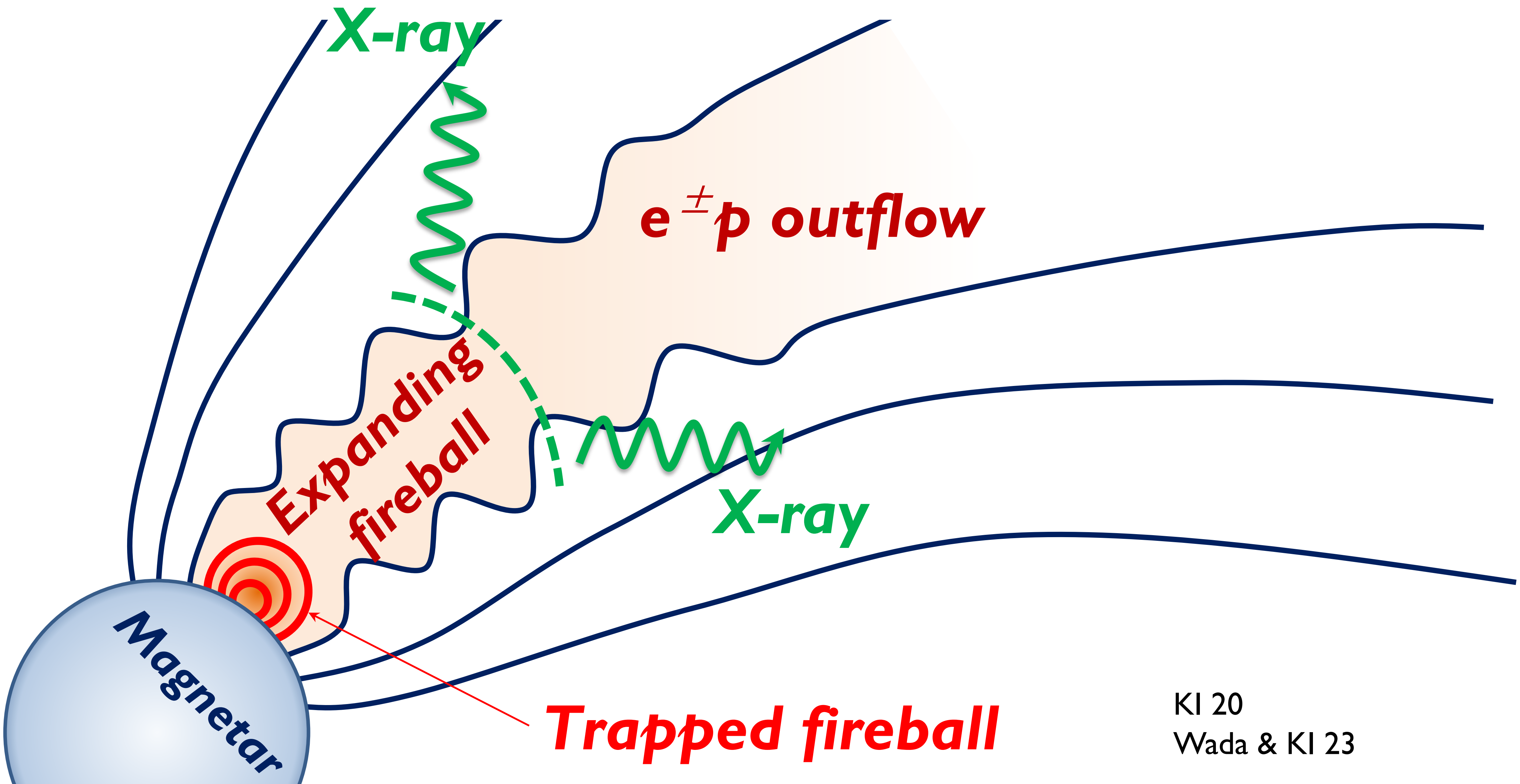
Summary

- ***Expanding fireball*** KI 20, Wada & KI 23
- ***Induced Compton scattering in B_0 for pairs***
- ***Ordinary, Charged & Neutral modes***
- ***Suppression of scatterings*** Nishiura, Kamijima, Iwamoto & KI 24; Ishizaki & KI 24
  - Gyroradius effect
  - Debye screening
- ***Facilitate FRB escape from a magnetosphere***
- ***Polarization $\perp B_0$ 100% to ~50%***

Fireball paradigm for coherent waves

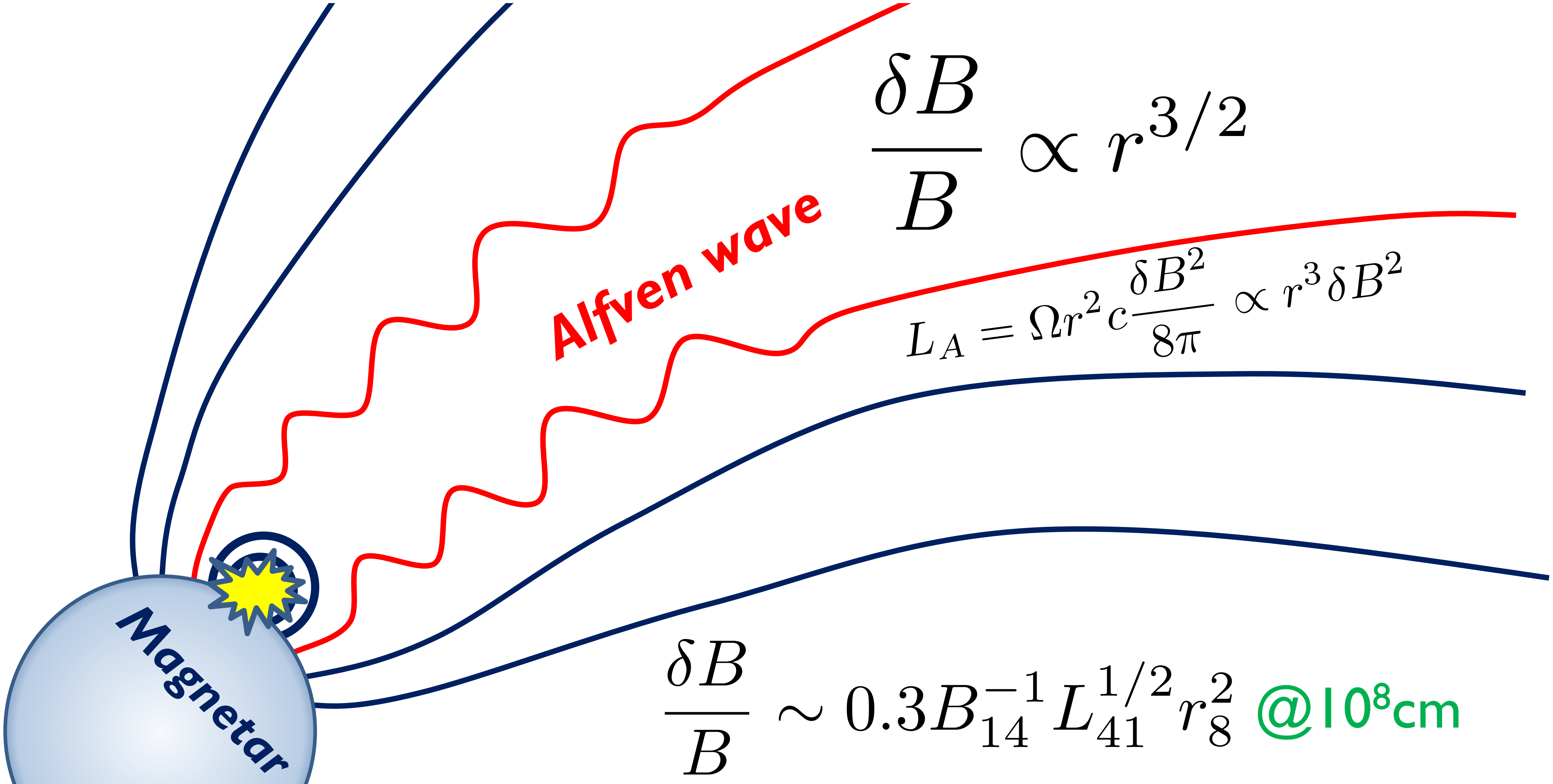
Thank You

X-ray from Fireball



KI 20
Wada & KI 23

Wave Amplitude



$$\frac{\delta B}{B} \propto r^{3/2}$$

$$L_A = \Omega r^2 c \frac{\delta B^2}{8\pi} \propto r^3 \delta B^2$$

$$\frac{\delta B}{B} \sim 0.3 B_{14}^{-1} L_{41}^{1/2} r_8^2$$ @10^8cm

Detail Calculations

Solution of density fluctuations

$$\widetilde{\delta n_\pm}(\boldsymbol{k},\omega)$$
$$= n_{\rm e0}\int {\rm d}^3\boldsymbol{v}\ \widetilde{\delta f_\pm}(\boldsymbol{k},\boldsymbol{v},\omega)$$
$$= -\frac{n_{\rm e0}}{m_{\rm e}}\left\{\widetilde{\phi_\pm}(\boldsymbol{k},\omega)\sum_{\ell=-\infty}^{+\infty}\int {\rm d}^3\boldsymbol{v}\frac{J_\ell^2\left(k_\perp r_{{\rm L}\pm}\right)\boldsymbol{k}\cdot\frac{\partial f_{0\pm}}{\partial \boldsymbol{v}^*}}{\omega-k_\parallel v_\parallel+\ell\omega_{{\rm c}\mp}}\right\}$$
$$\pm\frac{n_{\rm e0}H_\pm}{m_{\rm e}\varepsilon_{\rm L}}\left\{\widetilde{\phi_+}(\boldsymbol{k},\omega)\sum_{\ell=-\infty}^{+\infty}\int {\rm d}^3\boldsymbol{v}\frac{J_\ell^2\left(k_\perp r_{{\rm L}+}\right)\boldsymbol{k}\cdot\frac{\partial f_{0\pm}}{\partial \boldsymbol{v}^*}}{\omega-k_\parallel v_\parallel+\ell\omega_{{\rm c}-}}\right.$$
$$\left.-\widetilde{\phi_-}(\boldsymbol{k},\omega)\sum_{\ell=-\infty}^{+\infty}\int {\rm d}^3\boldsymbol{v}\frac{J_\ell^2\left(k_\perp r_{{\rm L}-}\right)\boldsymbol{k}\cdot\frac{\partial f_{0\pm}}{\partial \boldsymbol{v}^*}}{\omega-k_\parallel v_\parallel+\ell\omega_{{\rm c}+}}\right\},$$

longitudinal electric susceptibility

$$H_\pm\equiv\int {\rm d}^3\boldsymbol{v}\frac{4\pi e^2 n_{\rm e0}}{m_{\rm e}k^2}\sum_{\ell=-\infty}^{+\infty}\frac{J_\ell^2\left(k_\perp r_{{\rm L}\pm}\right)\boldsymbol{k}\cdot\frac{\partial f_{0\pm}}{\partial \boldsymbol{v}^*}}{\omega-k_\parallel v_\parallel+\ell\omega_{{\rm c}\mp}},$$

longitudinal dielectric constant

$$\varepsilon_{\rm L}(\boldsymbol{k},\omega)=1+H_+(\boldsymbol{k},\omega)+H_-(\boldsymbol{k},\omega).$$

Solution of EOM (v<<c)

$$\boldsymbol{v}_{0\pm}^{(1)}=\mp\frac{e}{m_{\rm e}c}\boldsymbol{A}_{0\parallel}\mp\frac{e}{m_{\rm e}c}\frac{\omega_0^2}{\omega_0^2-\omega_{\rm c}^2}\boldsymbol{A}_{0\perp}$$
$$-{\rm i}\frac{e}{m_{\rm e}c}\frac{\omega_0\omega_{\rm c}}{\omega_0^2-\omega_{\rm c}^2}\boldsymbol{A}_0\times\hat{\boldsymbol{B}}_0,$$

For thermal distributions

$$\sum_{\ell=-\infty}^{+\infty}\int {\rm d}^3\boldsymbol{v}\frac{J_\ell^2\left(k_\perp r_{\rm L}\right)\boldsymbol{k}\cdot\frac{\partial f_0}{\partial \boldsymbol{v}^*}}{\omega-k_\parallel v_\parallel-\ell\omega_{\rm c}}=\frac{m_{\rm e}k^2}{4\pi e^2 n_{\rm e0}}H_+$$
$$\underset{\sim}{\ell=0}\ \frac{2}{v_{\rm th}^2}\left\{1+\frac{\omega}{k_\parallel v_{\rm th}}I_0\left[\frac{1}{2}\left(\frac{k_\perp v_{\rm th}}{\omega_{\rm c}}\right)^2\right]\right.$$
$$\left.\times{\rm e}^{-\frac{1}{2}(k_\perp v_{\rm th}/\omega_{\rm c})^2}Z\left(\frac{\omega}{k_\parallel v_{\rm th}}\right)\right\}$$
$$\sim\frac{2}{v_{\rm th}^2}\left\{1+\frac{\omega}{k_\parallel v_{\rm th}}Z\left(\frac{\omega}{k_\parallel v_{\rm th}}\right)\right\}$$

Nonlinear Current

Nonlinear current as functions of $\widetilde{\phi_\pm}$ and $\boldsymbol{v}_{0\pm}^{(1)}$

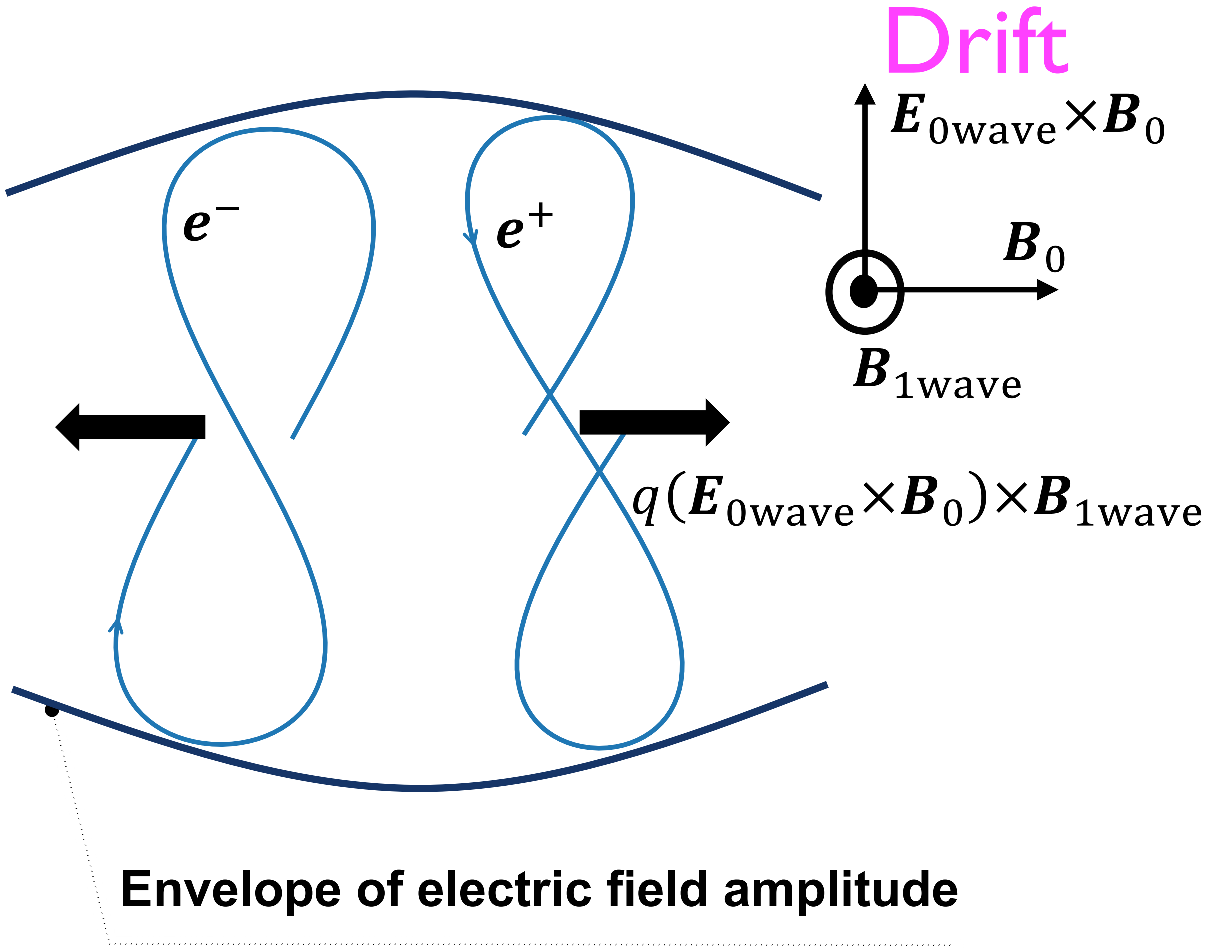
$$\widetilde{\boldsymbol{j}_1}^{*\mathrm{nonlinear}}(\boldsymbol{k}_1,\omega_1) = e\widetilde{\delta n_+}\,\boldsymbol{v}_{0+}^{(1)*} - e\widetilde{\delta n_-}\boldsymbol{v}_{0-}^{(1)*}$$

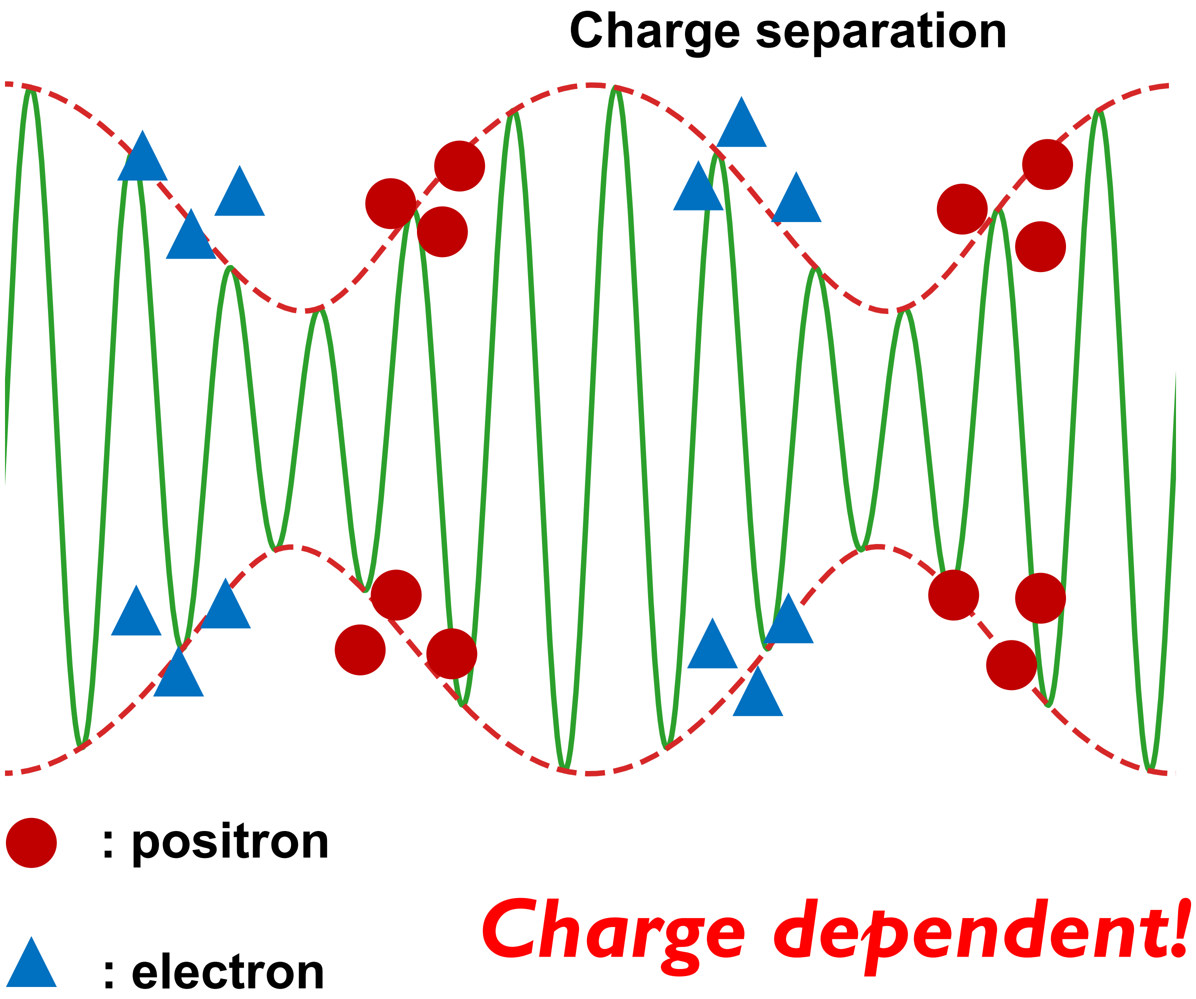
$$= (\cdots)\widetilde{\phi_+}\,\boldsymbol{v}_{0+}^{(1)*} + (\cdots)\widetilde{\phi_-}\,\boldsymbol{v}_{0+}^{(1)*} + (\cdots)\widetilde{\phi_-}\,\boldsymbol{v}_{0-}^{(1)*} + (\cdots)\widetilde{\phi_+}\,\boldsymbol{v}_{0-}^{(1)*}$$

ponderomotive potential	$\phi_\pm =$	$\frac{e^2}{2m_\mathrm{e}}\left\langle\frac{E_\parallel^2}{\omega_0^2}\right\rangle$	$-\frac{e^2}{2m_\mathrm{e}}\left\langle\frac{E_\perp^2}{\omega_\mathrm{c}^2-\omega_0^2}\right\rangle$	$\pm\mathrm{i}\frac{e^2}{2m_\mathrm{e}}\left\langle\frac{\omega_\mathrm{c}\left(E_z^*E_y-E_y^*E_z\right)}{\omega_0\left(\omega_\mathrm{c}^2-\omega_0^2\right)}\right\rangle$
fast velocity oscillation	$\boldsymbol{v}_{0\pm}^{(1)} =$	$\mp\frac{e}{m_\mathrm{e}c}\boldsymbol{A}_{0\parallel}$	$\mp\frac{e}{m_\mathrm{e}c}\frac{\omega_0^2}{\omega_0^2-\omega_\mathrm{c}^2}\boldsymbol{A}_{0\perp}$	$-\mathrm{i}\frac{e}{m_\mathrm{e}c}\frac{\omega_0\omega_\mathrm{c}}{\omega_0^2-\omega_\mathrm{c}^2}\boldsymbol{A}_0\times\widehat{\boldsymbol{B}}_0$
Excited mode		**Ordinary mode**	**Neutral mode**	**Charged mode**
Polarization of incident wave		**Parallel** $\boldsymbol{A}_{0\perp}=\boldsymbol{0}$	**Perpendicular** $\boldsymbol{A}_{0\parallel}=\boldsymbol{0}$	**Perpendicular** $\boldsymbol{A}_{0\parallel}=\boldsymbol{0}$

Ponderomotive Force in $B_0(\perp A_0)$

Charged mode

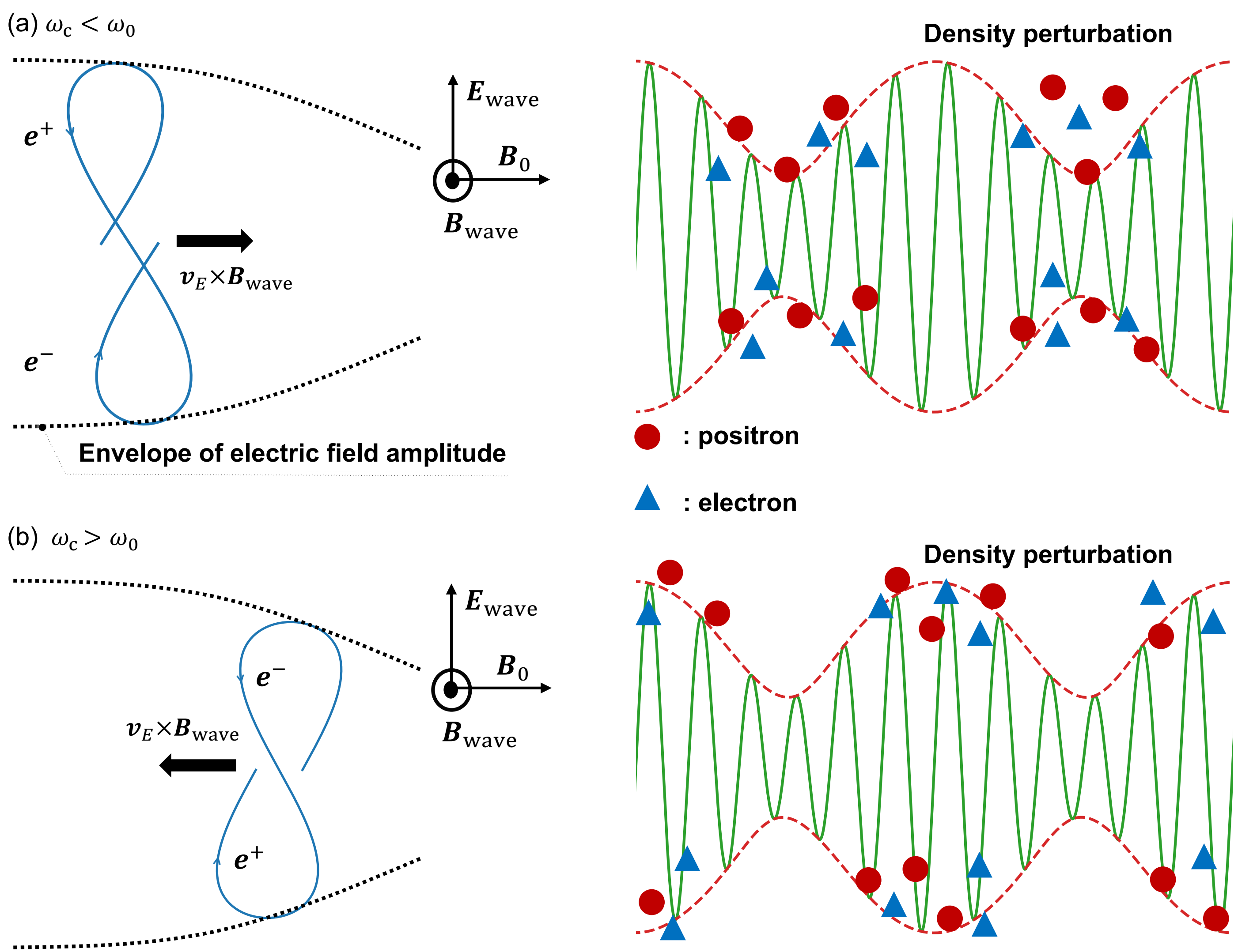




Charge dependent!

Ponderomotive Force in $B_0(\perp A_0)$

Neutral mode



Ponderomotive potential

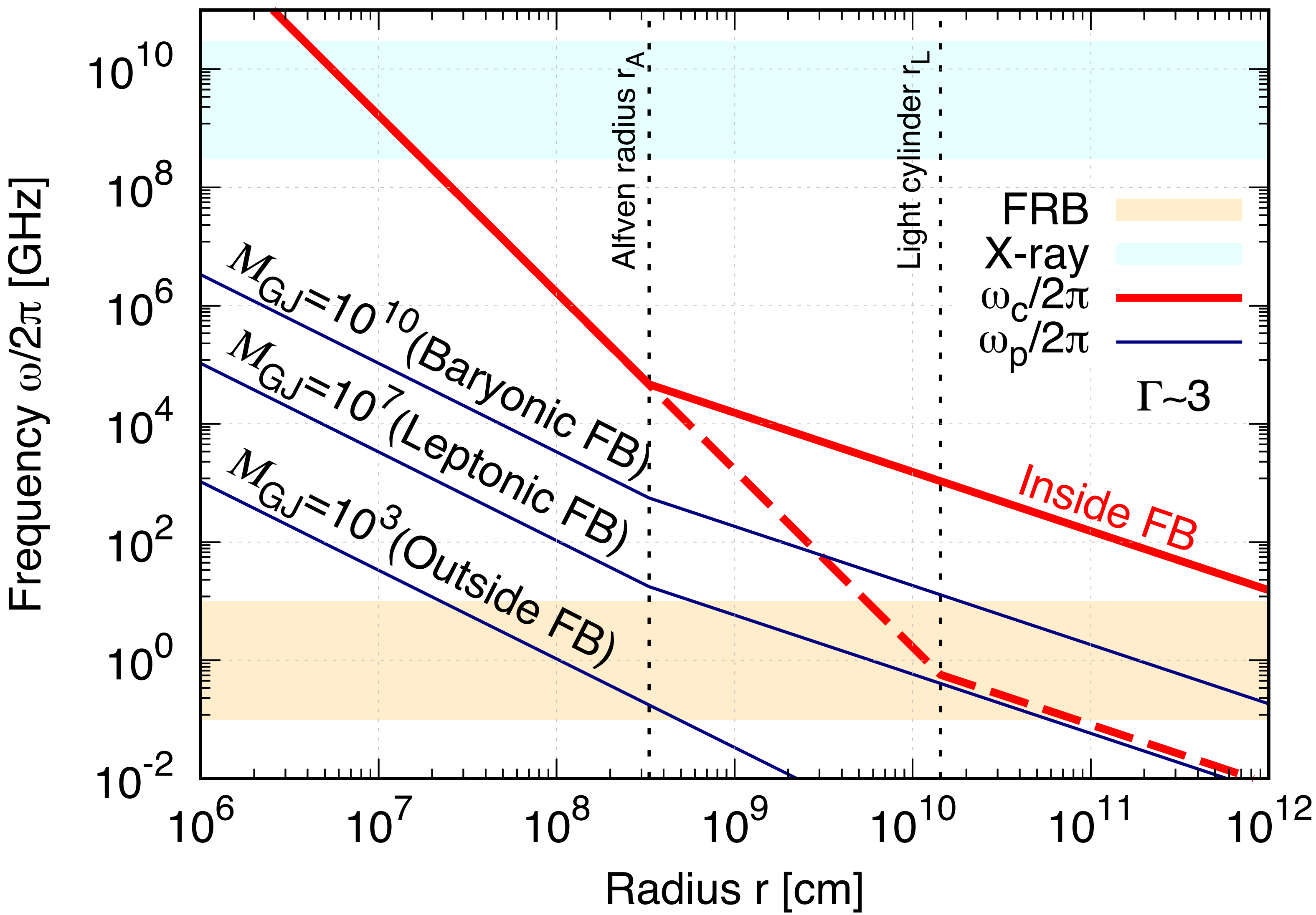
$$\phi_\pm = \frac{e^2}{2m_e}\left\langle \frac{E_\parallel^2}{\omega_0^2} - \underbrace{\frac{E_\perp^2}{\omega_c^2-\omega_0^2}}_{\text{Neutral mode}} + \mathrm{i}\underbrace{\frac{\omega_{c\pm}\left(E_z^* E_y - E_y^* E_z\right)}{\omega_0\left(\omega_c^2-\omega_0^2\right)}}_{\text{Charged mode}} \right\rangle$$

Neutral mode $\sim O(\omega_0/\omega_c)^2$

Charged mode $\sim O(\omega_0/\omega_c)$

Charge independent

Cyclotron & Plasma Frequency



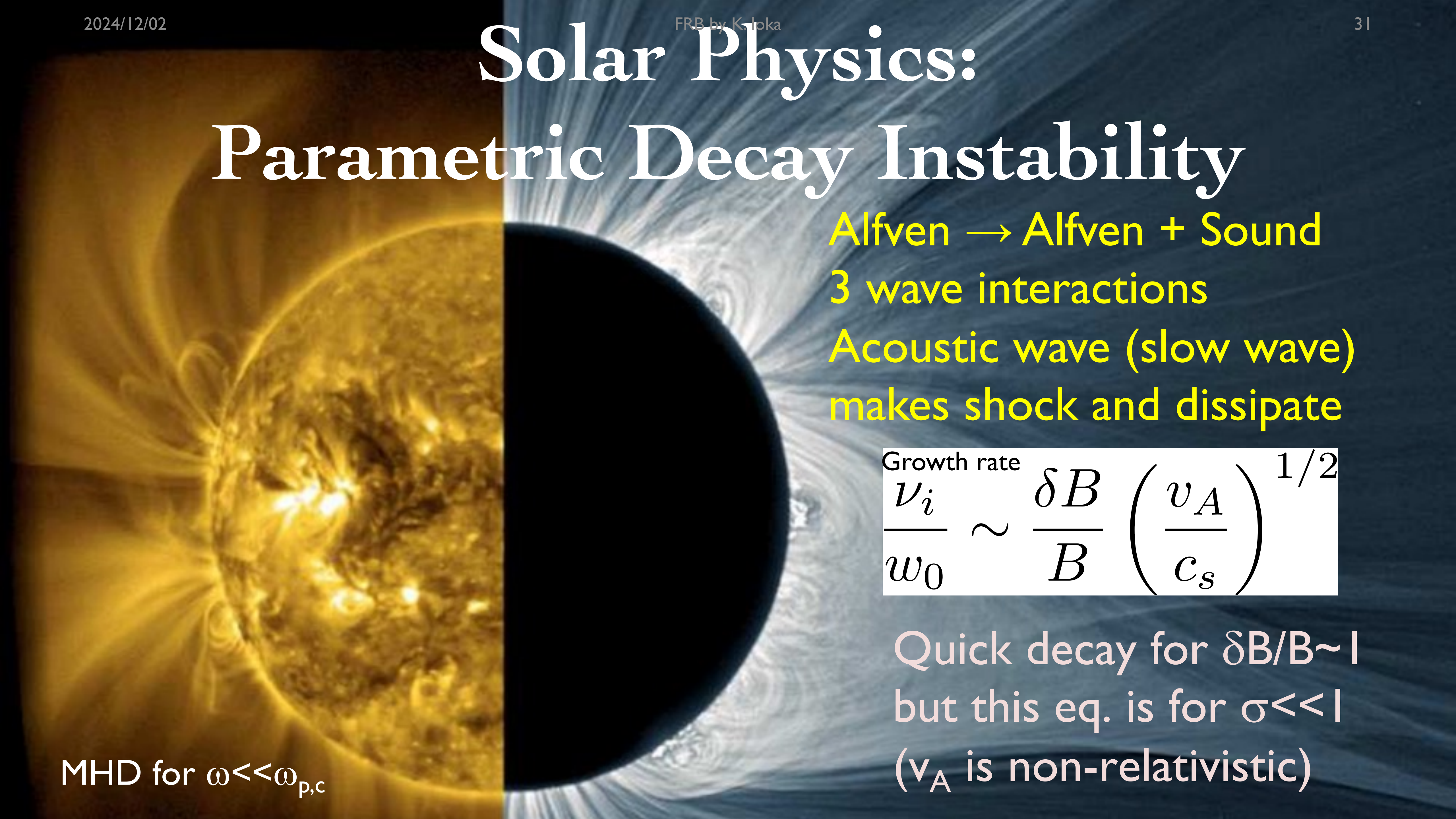
$$\omega_c = \Gamma \frac{qB}{mc}$$

$$\omega_p = \Gamma \left(\frac{4\pi q^2 n}{m} \right)^{1/2}$$

$\omega_c > \omega_{FRB}$

$\omega_p \sim \omega_{FRB}$

Solar Physics: Parametric Decay Instability



Alfven → Alfven + Sound

3 wave interactions

Acoustic wave (slow wave) makes shock and dissipate

Growth rate

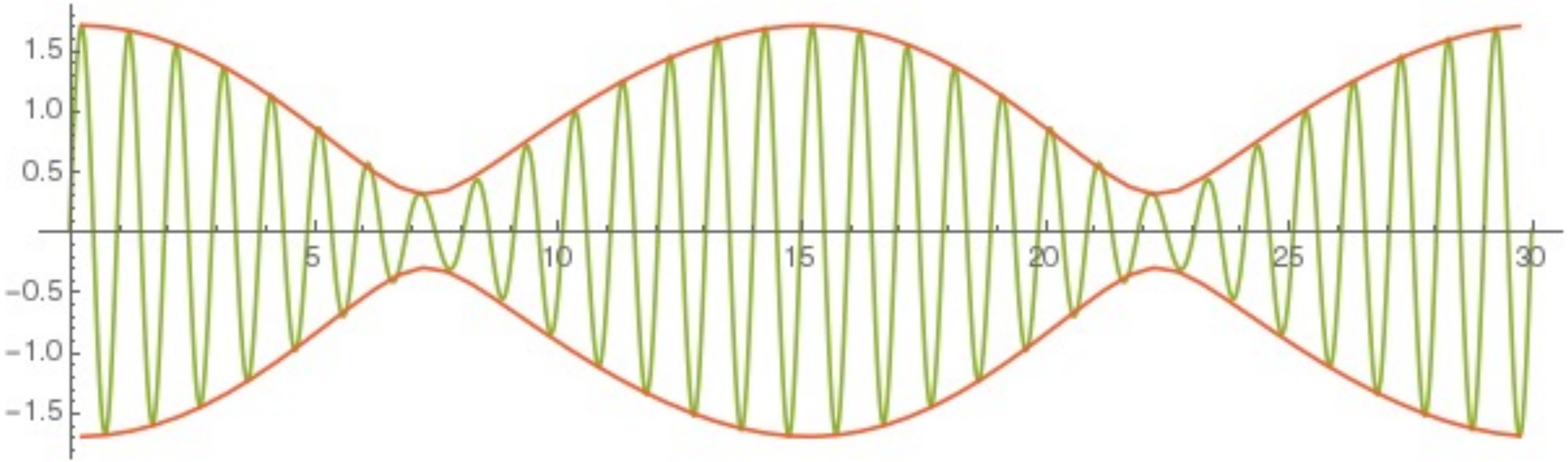
$$\frac{\nu_i}{w_0} \sim \frac{\delta B}{B} \left(\frac{v_A}{c_s} \right)^{1/2}$$

Quick decay for $\delta B/B \sim 1$
but this eq. is for $\sigma \ll 1$
(v_A is non-relativistic)

MHD for $\omega \ll \omega_{p,c}$

Alfven → Alfven + Sound

Parent Alfven + Daughter Alfven → Beat



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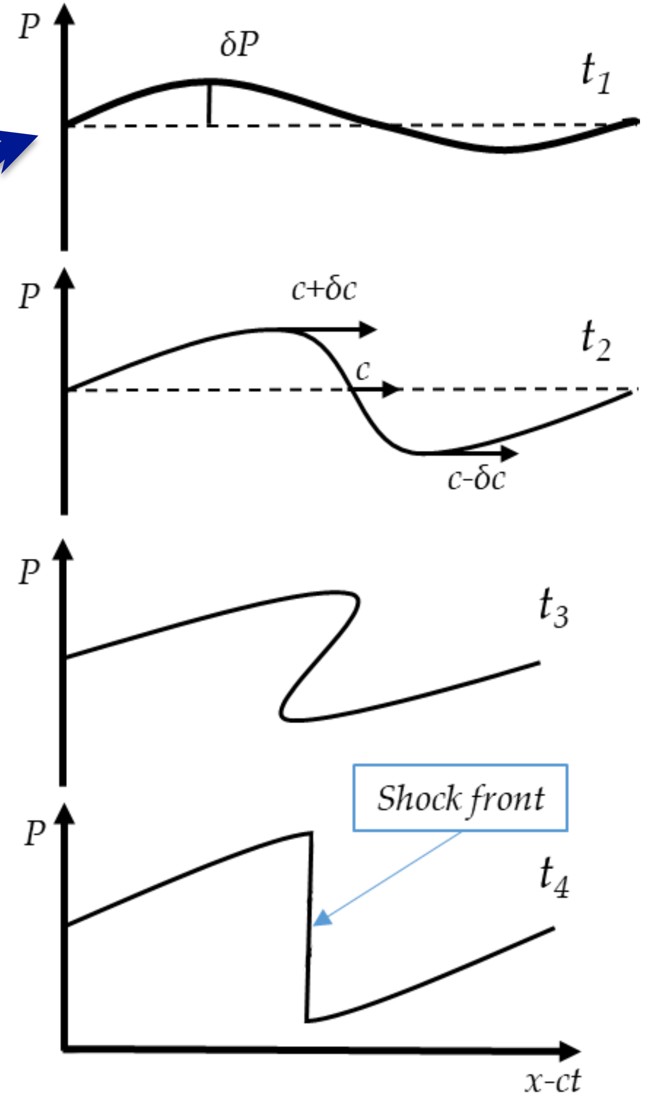
High & Low EM energy density → Sound wave

Induced Brillouin scattering

Sound Wave Dissipation

$A \rightarrow A + \underline{S}$



Effectively

$E_{\mathrm{Alfven}} \rightarrow E_{\mathrm{thermal}}$

Yevheniia Chernukha 19

High σ Limit

In the magnetosphere

$$\sigma \equiv \frac{B_0^2}{4\pi(\epsilon_0 + p_0)} \gg 1$$

B energy density >> Matter energy density

Force-free limit: Matter decouples from B

Alfven waves really decay?

To Relativistic MHD

Energy-momentum conservations & Induction eqs.

$$\frac{\partial}{\partial t}\left[(\epsilon+p)\gamma^2 - p + \frac{1}{8\pi}\left(E^2+B^2\right)\right] + \nabla\cdot\left[(\epsilon+p)\gamma^2 v + \frac{c}{4\pi}(\boldsymbol{E}\times\boldsymbol{B})\right] = 0$$

$$\frac{\partial}{\partial t}\left[(\epsilon+p)\,\gamma^2\boldsymbol{v} + \frac{c}{4\pi}(\boldsymbol{E}\times\boldsymbol{B})\right] + \nabla\cdot\left[(\epsilon+p)\gamma^2\boldsymbol{v}\otimes\boldsymbol{v} - \frac{c^2}{4\pi}(\boldsymbol{E}\otimes\boldsymbol{E}+\boldsymbol{B}\otimes\boldsymbol{B})\right] + c^2\nabla\left[p + \frac{E^2+B^2}{8\pi}\right] = 0$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla\times(\boldsymbol{v}\times\boldsymbol{B})$$

EOS

$$p_1 = \frac{C_s^2}{c^2}\epsilon_1, \quad C_s^2 = c^2\left(\frac{\partial p}{\partial \epsilon}\right)_s$$

1. Ideal MHD ($\omega << \omega_{p,c}$)
2. Adiabatic EOS
3. B_0 || k
4. Circular polarization
5. In the fluid comoving frame (background $v_{fluid} \sim 0$)

Perturbation

	Background	Parent wave	Daughter waves	
				Sound wave
$\boldsymbol{B} =$	$\boldsymbol{B}_0$	$+\,\delta\boldsymbol{B}$	$+\,\boldsymbol{b}_\perp$	
$\boldsymbol{v} =$		$\delta\boldsymbol{v}$	$+\,\boldsymbol{v}_\perp$	$+\,\boldsymbol{v}_\parallel$
$\epsilon =$	ϵ_0			$+\,\epsilon_\parallel$
		Alfvén wave	Alfvén wave	
	$O(1)$	$O(\eta)$	$O(\varepsilon)$	

Ishizaki & KI 24

Do daughter waves grow?

Perturbed Equations

Ishizaki & KI 24

$$\frac{1}{c}\frac{\partial e_\parallel}{\partial t}+\beta_s\frac{\partial u_\parallel}{\partial z}=-\frac{\sigma}{1+\sigma}\frac{1}{c}\frac{\partial}{\partial t}\left(\delta\boldsymbol{u}\cdot\boldsymbol{u}_\perp\right)-\sigma\delta\boldsymbol{u}\cdot\left(\frac{1}{c}\frac{\partial \boldsymbol{u}_\perp}{\partial t}-\beta_A\frac{\partial \boldsymbol{e}_\perp}{\partial z}\right) \tag{27}$$

$$\frac{1}{c}\frac{\partial u_\parallel}{\partial t}+\beta_s\frac{\partial e_\parallel}{\partial z}=-\theta^{-1}\beta_A\frac{\partial}{\partial z}\left(\delta\boldsymbol{e}\cdot\boldsymbol{e}_\perp\right)+\sigma\theta^{-1}\delta\boldsymbol{e}\cdot\left(\frac{1}{c}\frac{\partial \boldsymbol{u}_\perp}{\partial t}-\beta_A\frac{\partial \boldsymbol{e}_\perp}{\partial z}\right) \tag{28}$$

$$\frac{1}{c}\frac{\partial \boldsymbol{u}_\perp}{\partial t}-\beta_A\frac{\partial \boldsymbol{e}_\perp}{\partial z}=\theta\beta_A^2\frac{1}{c}\frac{\partial}{\partial t}\left(u_\parallel\delta\boldsymbol{e}\right)-\frac{1}{1+\sigma}\left[\beta_s u_\parallel\frac{\partial}{\partial z}(\delta\boldsymbol{u})+\beta_A e_\parallel\frac{\partial}{\partial z}(\delta\boldsymbol{e})+\beta_s^2\frac{1}{c}\frac{\partial}{\partial t}\left(e_\parallel\delta\boldsymbol{u}\right)\right] \tag{29}$$

$$\frac{1}{c}\frac{\partial \boldsymbol{e}_\perp}{\partial t}-\beta_A\frac{\partial \boldsymbol{u}_\perp}{\partial z}=-\theta\beta_A\frac{\partial}{\partial z}\left(u_\parallel\delta\boldsymbol{e}\right) \tag{30}$$

For the normalized quantities

$$\delta\boldsymbol{u}\equiv\frac{\delta\beta}{\beta_A},\quad \boldsymbol{u}_\perp\equiv\frac{\beta_\perp}{\beta_A}$$

$$\delta\boldsymbol{e}=\frac{\delta\boldsymbol{B}}{B_0},\quad \boldsymbol{e}_\perp=\frac{\boldsymbol{b}_\perp}{B_0}$$

$$u_\parallel\equiv\frac{\beta_\parallel}{\beta_s},\quad e_\parallel\equiv\frac{\epsilon_\parallel}{w_0}$$

Alfven velocity

$$\beta_A^2=\sigma/(1+\sigma)$$

Enthalpy

$$w_0\equiv\epsilon_0+p_0$$

Dimensionless parameters

$$\sigma\equiv\frac{B_0^2}{4\pi\left(\epsilon_0+p_0\right)}$$

$$\theta\equiv\frac{\beta_s}{\beta_A}$$

Dispersion Relation

Ishizaki & KI 24

Fourier mode expansion

Parent
$$\delta \boldsymbol{e} = \frac{1}{\sqrt{2}}\left(\delta e_0 \exp\left(i\phi_0\right)\mathbf{e}_j + \text{ c.c. }\right)$$
$$\delta \boldsymbol{u} = -\delta \boldsymbol{e}$$

Sound
$$e_{\parallel} = \frac{1}{2}\left(e_k \exp(i\phi) + \text{ c.c. }\right), \quad u_{\parallel} = \frac{1}{2}\left(u_k \exp(i\phi) + \text{ c.c. }\right)$$

Alfven
$$\boldsymbol{e}_{\perp} = \frac{1}{\sqrt{2}}\left(e_+ \exp\left(i\phi_+\right)\mathbf{e}_j + \text{ c.c. }\right) + \frac{1}{\sqrt{2}}\left(e_- \exp\left(i\phi_-\right)\mathbf{e}_j + \text{ c.c. }\right)$$
$$\boldsymbol{u}_{\perp} = \frac{1}{\sqrt{2}}\left(u_+ \exp\left(i\phi_+\right)\mathbf{e}_j + \text{ c.c. }\right) + \frac{1}{\sqrt{2}}\left(u_- \exp\left(i\phi_-\right)\mathbf{e}_j + \text{ c.c. }\right)$$

$$\phi_0 \equiv k_0 z - \omega_0 t$$
$$\phi \equiv kz - \omega t$$
$$\phi_+ \equiv \phi_0 + \phi, \quad \phi_- \equiv \phi_0 - \phi$$

(satisfying resonance conditions)

Det (6 ×6 matrix) = 0 → Dispersion relation

$$(\omega - k)^2\left(\omega^2 - \theta^2 k^2\right)\left\{(\omega + k)^2 - 4\right\} = \frac{1}{(1+\sigma)^4}\eta^2(\omega - k)\left(S_0 + S_1\sigma + S_2\sigma^2 + S_3\sigma^3 + S_4\sigma^4\right)$$

$$S_0 = k^2\left(\omega^3 + k\omega^2 - 3\omega + k\right)$$

$S_1=\ldots$, $S_2=\ldots$, $S_3=\ldots$, $S_4=\ldots$

(in the unit of $k_0=1$, $w_0=1$)

Decay Rate

Ishizaki & KI 24

$$\frac{\mathrm{Im}\,\delta\omega}{\omega_0} \sim \frac{1}{2}\eta\theta^{-1/2}\sigma^{-1/2} \sim \underbrace{\frac{1}{2}\frac{\delta B}{B}\left(\frac{v_A}{c_s}\right)^{1/2}}_{\text{Non-rela}} \underbrace{\sigma^{-1/2}}_{\text{Rela}}$$

Non-rela

Rela

$$\eta \sim \frac{\delta B}{B}$$:Wave amplitude of Alfven wave

$$\sigma \equiv \frac{B_0^2}{4\pi(\epsilon_0 + p_0)}$$: Energy density ratio
"sigma" parameter σ >> 1

$$\theta \equiv \frac{\beta_s}{\beta_A}$$ ~ sound velocity/c

Alfven wave velocity
$\beta_A^2 = \sigma/(1+\sigma)$ ~1

Nonlinear Interaction

BG — Parent Alfven — Daughter Alfven — Daughter sound wave

$$\mathbf{B} = \mathbf{B}_0 + \delta\mathbf{B}_\perp(z,t) + \mathbf{b}_\perp(z,t),$$
$$\mathbf{V} = \delta\mathbf{V}_\perp(z,t) + \mathbf{v}_\perp(z,t) + \mathbf{v}_\parallel(z,t),$$
$$\rho = \rho_0 + \rho(z,t).$$

Parent Alfven: Circular

$$\delta\mathbf{B}_\perp(z,t) = \delta\mathbf{B}_\perp \exp[i(k_0 z - \omega_0 t)] + c.c.,$$
$$\delta\mathbf{V}_\perp = -\frac{B_0}{4\pi\rho_0}\frac{k_0}{\omega_0}\delta\mathbf{B}_\perp(z,t),$$
$$\omega_0^2 = (B_0^2/4\pi\rho_0)k_0^2 = V_A^2 k_0^2$$

Ideal MHD, $B_0 \parallel k$

$$\frac{\partial v_\parallel}{\partial t} + \frac{c_s^2}{\rho_0}\frac{\partial \rho}{\partial z} = -\frac{\partial}{\partial z}\left(\frac{\mathbf{b}_\perp\cdot\delta\mathbf{B}_\perp}{4\pi}\right)\frac{1}{\rho_0},$$

nonlinear dominant term

$$\frac{\partial \rho}{\partial t} + \rho_0\frac{\partial v_\parallel}{\partial z} = 0,$$

$$\frac{\partial \mathbf{v}_\perp}{\partial t} - \frac{B_0}{\rho_0}\frac{\partial}{\partial z}\left(\frac{b_\perp}{4\pi}\right) = -v_\parallel\frac{\partial}{\partial z}(\delta V_\perp) - \frac{B_0\rho}{4\pi\rho_0^2}\frac{\partial}{\partial z}(\delta\mathbf{B}_\perp)$$

nonlinear dominant term

$$\frac{\partial \mathbf{b}_\perp}{\partial t} - B_0\frac{\partial}{\partial z}\mathbf{v}_\perp = -\frac{\partial}{\partial z}(v_\parallel\delta\mathbf{B}_\perp).$$

Resonant conditions

$$\omega_A = \omega_s + \omega_0,$$
$$k_A = k_s + k_0,$$

$$\frac{\partial v_\parallel}{\partial t} = i(k_A - k_0)\frac{B_0 k_A}{\rho_0\omega_A}\left(\frac{\delta\mathbf{B}_\perp^*\mathbf{v}_\perp}{4\pi}\right),$$
$$\frac{\partial \mathbf{v}_\perp}{\partial t} = -\frac{iB_0 k_s k_0}{4\pi\rho_0\omega_s}v_\parallel\delta\mathbf{B}_\perp,$$

$$v_\parallel, v_\perp \sim e^{i\nu t} \quad \rightarrow \textbf{Im } \nu$$

Galeev & Oraevskii 63; Sagdeev & Galeev 69

Non-relativistic Limit

Neglecting $O(\beta_A{}^2)$ $\sigma = \beta_A^2/(1-\beta_A^2)$ is small

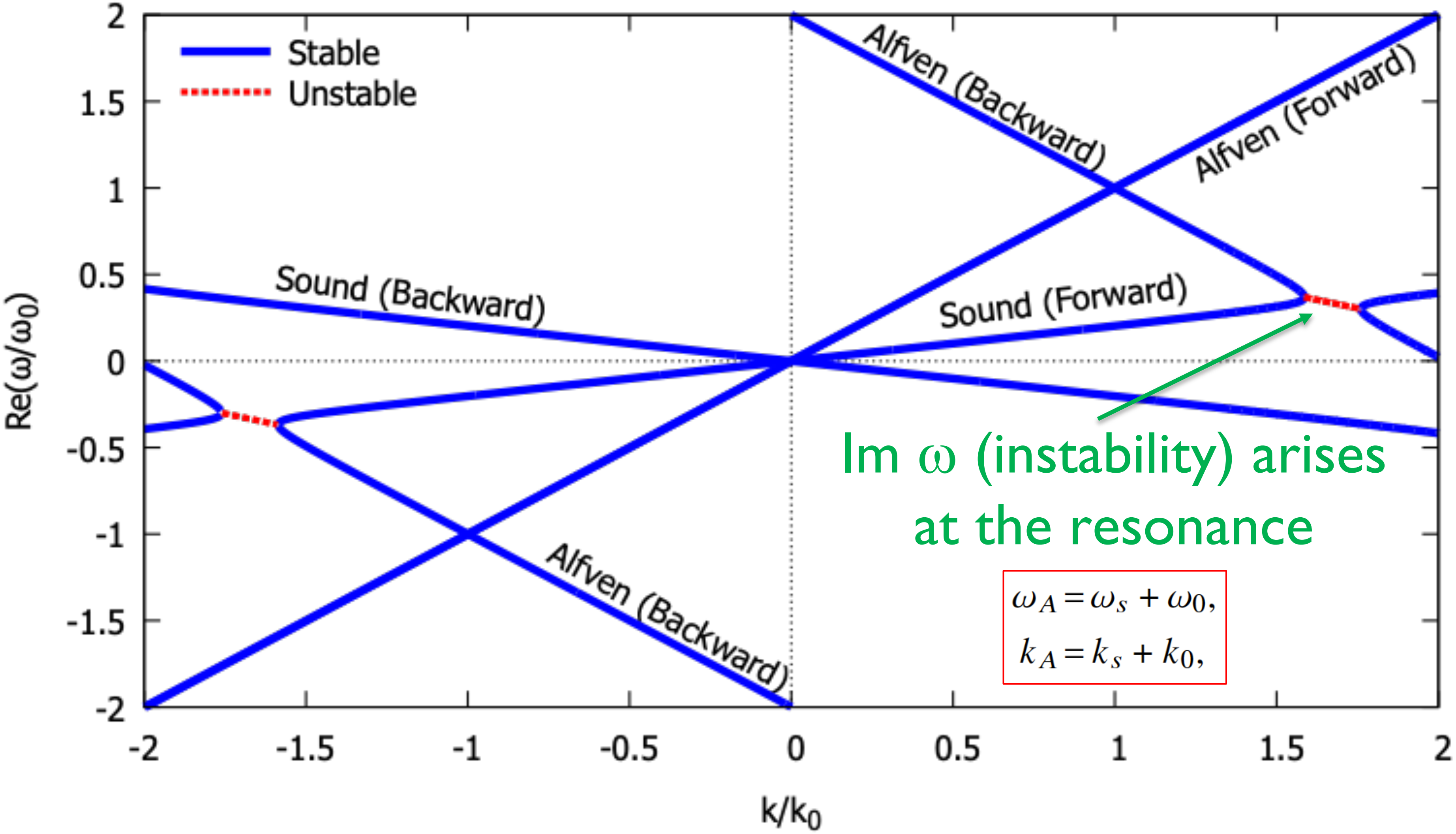
$$(\omega - k)^2 \left(\omega^2 - \theta^2 k^2\right) \left\{(\omega + k)^2 - 4\right\} = \eta^2 k^2 (\omega - k) \left(\omega^3 + k\omega^2 - 3\omega + k\right)$$

the same as Goldstein (1978) & Derby (1978)

In the limit $\beta = c_s{}^2/v_A{}^2 \to 0$, the decay instability is recovered:
forward Alfven → forward acoustic + backward Alfven

$$\frac{\omega_i}{\omega_0} = \frac{\delta B}{B} \left(\frac{v_A}{c_s}\right)^{1/2}$$

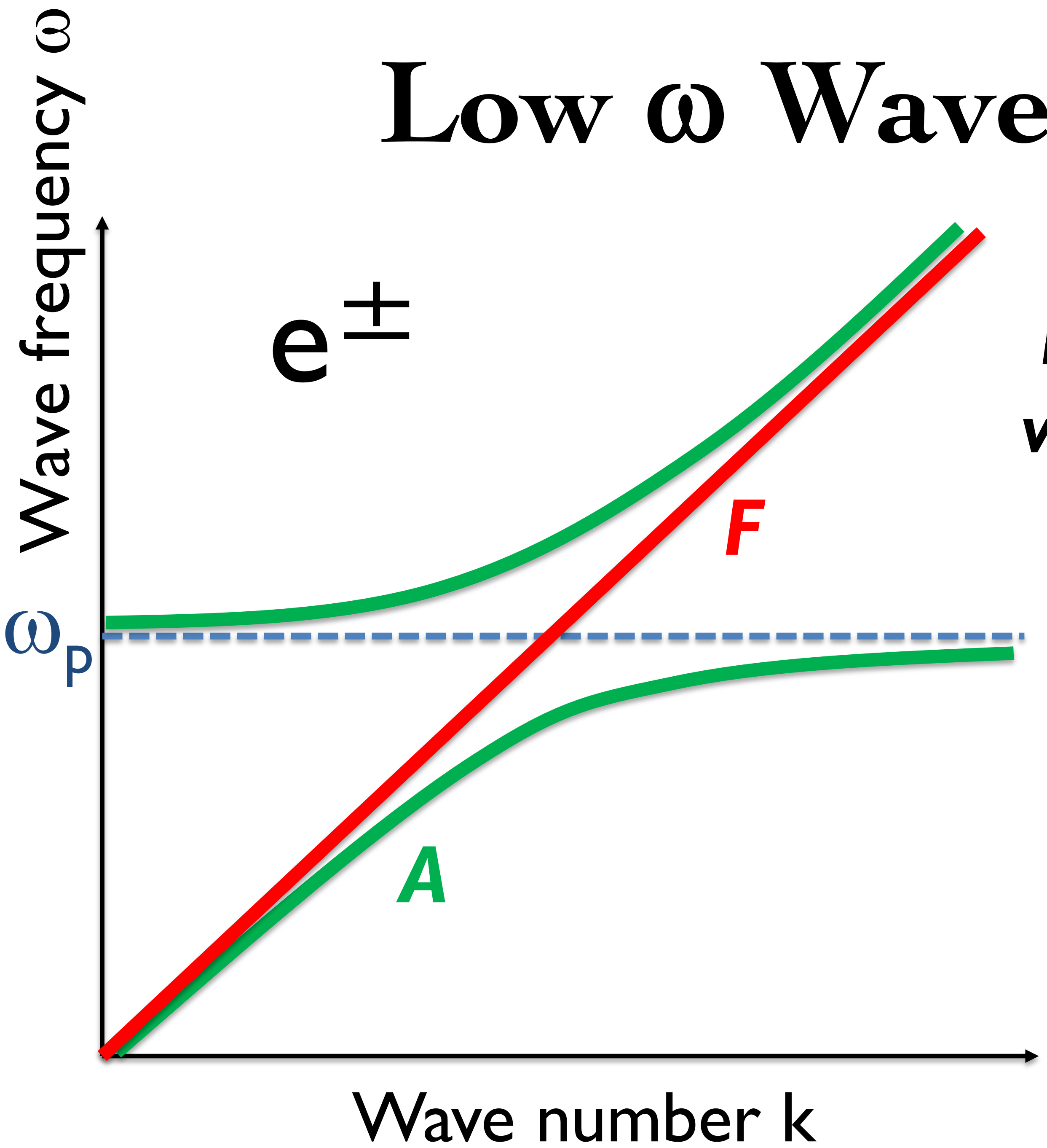
Dispersion Relation

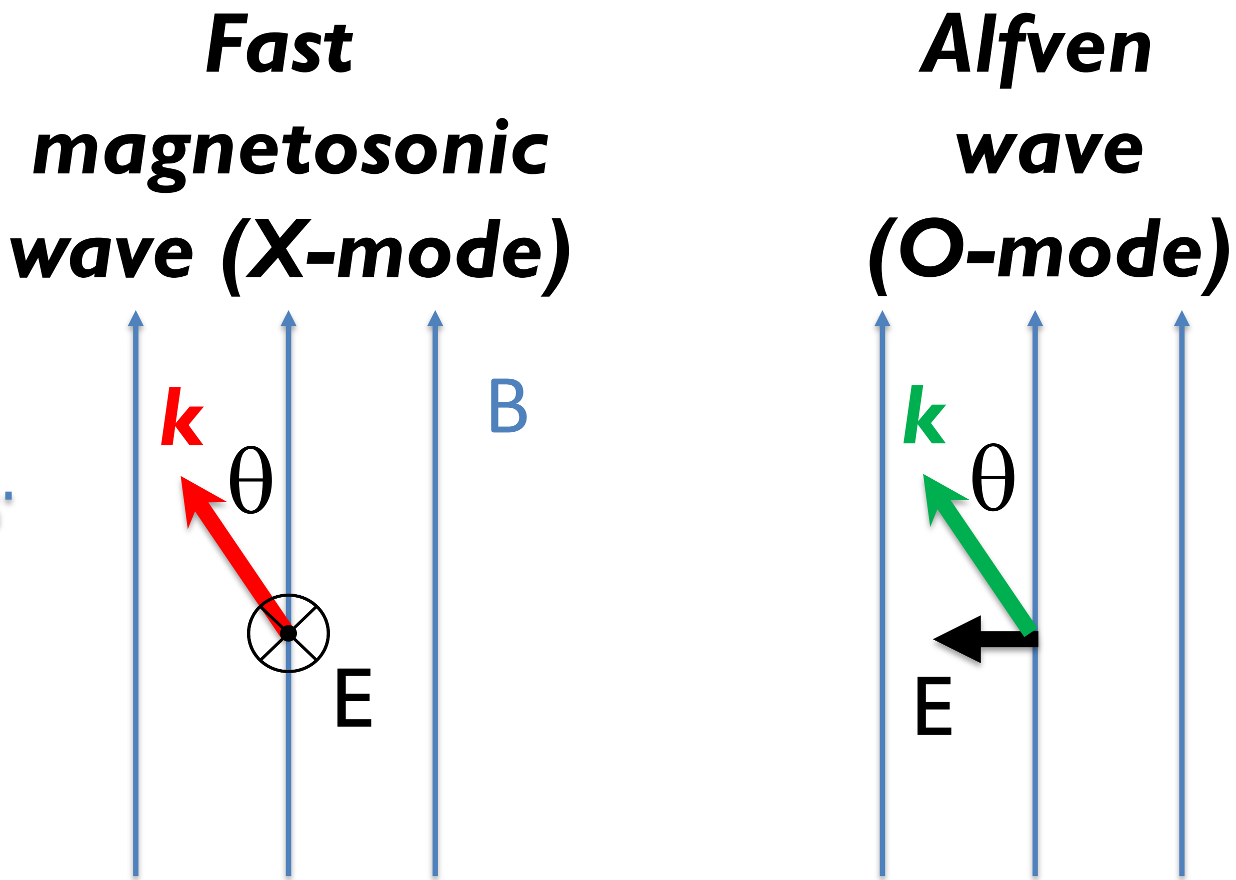


Im ω (instability) arises at the resonance

$$\omega_A = \omega_s + \omega_0,$$
$$k_A = k_s + k_0,$$

Low ω Waves in Strong B





$A+A \leftrightarrow F$

$A+F \leftrightarrow F$