The influence of magnetic vortices on physical observables in the gluon plasma

(1) Thermal gluon propagators in the infrared region (Phys. Rev. D83:114501,2011),
(2) a trial for transport coefficients and
(3) equation of state of the gluon plasma via the center (magnetic) vortex mechanism.

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Contents of my talk

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Motivation: Strongly-interacting quark-gluon plasma (sQGP)
Quark-gluon plasma (1)

◆ **Quark-gluon plasma (QGP)**
  - Heavy-ion experiment (RHIC, 2004) and produces a quark-gluon plasma (QGP); also LHC on Nov. 2010 reports producing it at (maybe) higher temperature.
  - Hydro calc. works very well, zero-shear viscosity (nearly perfect fluid), and jet quenching, etc.; small shear viscosity, also by lattice simulations.
  - Picture of *strongly-interacting QGP (sQGP)* is established now.
  - But, we have big question: what is *Physics of sQGP*?

◆ Singular behavior from the lattice we'd like to focus on here:
  - From lattice calculations: *non-zero spatial-string tension, magnetic masses, magnetic degrees of freedom are so singular in QGP.* (?)
Thermal magnetic monopoles and vortices?

Interesting idea: Magnetic plasma made of monopoles and/or center vortices:


II. Chernodub and Zakharov, PRL98(2007)082002. Degrees of freedom of center (magnetic) vortices have been introduced by elucidating the sQGP physics.

III. Chernodub, Nakamura, Zakharov, PRD78,074021(2008). Vortices provide a large contribution to eos of sQGP.

Here, we investigate how the vortices affect color-screened gluons, transport coefficients, eos, etc.
Spatial Wilson loop in the QGP

- Spatial-Wilson loop (not extending to temporal dimension) gives a linearly rising potential in the QGP phase. Note that the Polyakov line correlator (wrapping to temporal dimension) gives a screened (non-confining) potential.

\[ W(R, S) \sim \exp(-\sigma_s RS) \]

\[ \sqrt{\sigma_s(T)} = cg^2(T)T \]

FIG. 1. The pseudopotentials \( V_T(R) \) minus the (constant) self-energy contributions \( V_0 \) [Eq. (4)] on lattices of size \( N_x \times 32^3 \) for \( \beta = 2.74 \) as a function of the spatial separation \( R \) measured in lattice units.

FIG. 3. The ratio of the critical temperature and square root of the spatial string tension versus temperature for \( \beta = 2.74 \). The line shows a fit to the data in the region \( 2 \leq T/T_c \leq 8 \) using the two-loop relation for \( g(T) \) given in Eq. (7).

G.S. Bali, et. al, PRL71,3059(1993)
Screened potentials in the QGP

Screening potentials between two quarks in each color-channel:

Screening masses in the QGP

Definition and role of center vortices below $T_c$
Center vortices on the lattice

- **Center (magnetic) vortices** (as a topological defect of SU(N) gauge theory) are responsible for non-perturbative phenomena such as color confinement and chiral symmetry breaking. (t’Hooft, Mack, Cornwall, etc.)
- Center vortices are defined via the center group $Z(N)$ of the gauge group $SU(N)$.

$$
\pi_1(SU(N)/Z(N)) = Z(N)
$$

- Illustration of vortex-monopole chain; *Chernodub, et al*, *PRD78:074021, 2008*
Maximal center projection (1)

◆ Numerical technique
  □ Direct Maximal Center Projection (MCP) by Debbio, et. al, PRDv58,094501(1999)

◆ We apply the MCP to all configurations of the SU(2) gauge field

All the Us ⇒ ±I

Maximize $R = \frac{1}{VT} \sum_{x,t} \text{Tr}[U_\mu(x,t)]^2$

Example of vortices on 2D lattice for SU(2)
Maximal center projection (2)

- Removing center vortex, via de Forcrand – D’Elia procedure, PRL82, 4582 (1999):

\[ Z_\mu(x) = \text{sgn} \text{ Tr}[U_\mu(x)] \]

\[ U_\mu(x) \rightarrow U'_\mu(x) = Z_\mu(x)U_\mu(x) \]

- Color confinement disappears and chiral symmetry restores.

- Vortices carry the non-perturbative IR physics of QCD
- Handling vortices numerically enables us to switch on/off non-perturbative mode!!

In particular, this technique shall be applied to the QGP physics.
Linearly rising quark potential

◆ Removing vortices eliminates confinement.
SU(2), 12

Chiral symmetry breaking

◆ Removing vortices restores chiral symmetry

Relevance of Center Vortices to QCD; Forcrand and D’Elia, PRL82,4582(1999)

Note that however, by the recent analyses, this depends on choice of lattice action
More refs. for center vortices

This picture has been strongly supported by lattice simulations:

- Identify center vortices on the lattice; Debbio, et. al, PRD58, 094501(1999).
- Removing center vortices eliminates confinement and restores chiral symmetry; de Forcrand, D’Elia, PRL82,4582(1999)
- Vortex density shows asymptotic scaling; Langfeld, PRD69,(2004)014503; Langfeld, et.al, PLB419(1998)317
Numerical results: Electric and magnetic gluon propagators

M.N. Chernodub, Y. Nakagawa, A. Nakamura, T. Saito, V.I. Zakharov,
Gluon propagators at finite temperature

- Color-screened gluon propagator at finite temperature
  - Lorentz (O(4)) invariance broken due to the shrinking temporal dimension.
  - Electric and magnetic gluons are screened differently.

- Electric (color-Debye) mass
  - Leading order perturbation
    \[ m_E \sim g(T)T \]
  - Nonperturbative method (by Poylakov line)
    \[ -\log \langle P(R)P^+(0) \rangle = V(R) \sim C_r \frac{\exp(-m_ER)}{R} \]
  - Important for screening effect.

- Magnetic mass
  - Magnetic gluon also screened for non-abelian gauge theory (not for U(1))
  - But, we can not calculate consistently the magnetic mass by the perturbation.
    ---> Linde divergence problem (Linde, PLB96,289,1980)
  - But, the prediction of 3d argument gave us
    \[ m_M \sim g^2(T)T \]
  - Lattice computations show non-zero magnetic masses at finite temperature.
Gluon propagators drastically change in infrared regions, in particular for magnetic sector, after removal of center vortices.

Similar results from gluon propagators in the confinement regions; Gattnar, et. al. PRL93(2004)061601
At higher temperature, the same results are obtained.
Gribov-Zwanziger confinement scenario for the Coulomb gauge QCD survives in QGP.
( Greensite, et. al, PRD67, 094503 (2003); PRD69, 074506 (2004); Nakagawa, et. al, PRD73 (2006) 094504 )

- Time-time (electric) propagator is singular in the infrared limit.
  - Instantaneous interaction and non-zero string tensions
Example for center vortex removal

◆ Numerical study of Coulomb gauge QCD via center vortex (Greensite, Olejnik, Zwanziger, PRD69, 074506 (2004) )

• Gribov-Zwanziger scenario in the Coulomb gauge QCD: Instantaneous interaction (link-link correlator on the lattice ) produces a confining potential even in the QGP phase.

\[ V(R,0) = \log(U(R,0)U^+(0,0)) \]

\[ T/T_c \sim 1.40 \]

- Large difference, because this is magnetic.
- Small difference, because this is electric.

**Color instantaneous potential**
(magnetic and spatial)

**Color -Debye potential**
(electric)
Numerical results: Transport coefficients

Preliminary and on-going calculations
Transport Coefficient

◆ Formulation of transport coefficients coming from linear response theory.
◆ Hydro-model analyses shows ideal gas, zero-shear viscosity.
◆ Frequently exchanging momentum among elements; short mean-free path.
◆ Perturbative expression for the viscosity is

\[
\frac{\eta}{T^3} = \frac{\kappa}{g^4 \log(g^{-1})}
\]

\[
\kappa = \begin{cases} 
27.126(N_f = 0) \\
86.473(N_f = 2)
\end{cases}
\]

\[
g \to 0; \eta \to \text{large} \\
g \to \infty; \eta \to \text{zero; perfect liquid}
\]

◆ sQGP picture has been established but why it is so small?

Formulation of transport Coefficient

◆ Energy-momentum tensor

\[ T_{\mu \nu} = 2Tr(F_{\mu \sigma}F_{\nu \sigma} - \frac{1}{4} \delta_{\mu \nu} F_{\rho \sigma}F_{\rho \sigma}) \]
\[ T_{\mu \nu} = 0 \quad U_{\mu \nu}(x) = \exp(ia^2 gF_{\mu \nu}(x)) \]

◆ Green function on shear viscosity

\[ \eta = -\int d^3x \int_{-\infty}^{t} dt e^{i(t_1 - t)} \int_{-\infty}^{t_1} dt' \left\langle T_{12}(x,t)T_{12}(x',t') \right\rangle_{ret} \]

◆ However, we can only calculate thermal Green function on the Euclidean lattice and have to find retarded Green function, which is a very hard task. So usually we assume the following spectral function,

\[ \rho = \frac{A}{\pi} \left( \frac{\gamma}{(m - \omega)^2 + \gamma^2} + \frac{\gamma}{(m + \omega)^2 + \gamma^2} \right) \]

◆ and, for instance, on the shear viscosity,

\[ \eta = 2A \frac{2\gamma m}{(\gamma^2 + m^2)^2}, \text{ etc.} \]

Correlators of shear viscosity

- Transport coefficients are also affected by removing vortices from the original lattice?

Left fig: Thermal correlator of energy-momentum tensor for the shear viscosity after/before center removal has been calculated.

It feels vortices !!!

But it needs more statistics and can not determined properly yet.
Numerical results: Equation of State of the gluon plasma

Preliminary and on-going calculations
Equation of State of QGP (1)

Equation of state of QGP has been calculated a lot in the lattice; quenched and unquenched cases at finite temperature with many type of lattice actions and recently a challenging finite-density studies are on progress.

But, lattice calculations do not touch Stefan Boltzman limit (SBL) even at high temperature. Problem (?)

Simply, this is a proof that QGP still strongly interacts?

Examples of the pressures (quenched case)

M. Okamoto, et. al, (CP-PACS Coll.) PRD60 (1999) 094510
Equation of State of QGP (2)

Equation of state in the integral method on the lattice

\[ T = 1/(N_t a), \quad V = (N_s a)^3 \]

For large and homogeneous system,

\[ p = -f = \frac{T}{V} \ln Z(T,V) \]

\[ \frac{p}{T^4} \bigg|_{\beta_0}^{\beta} = \int_{\beta_0}^{\beta} d\beta' \Delta \quad \Delta = N_\tau^4(S_\tau - S_0) \]

\[ \Delta = N_{\tau}^4 \left( S_{\tau} - S_0 \right) \]

**Normal**

**Removal**

**Lattice:**

\[ 16^3 \times 4; \ \beta = 2.30 - 2.90; \ \beta_c \approx 2.30; \]

**Heat bath:**

\[ \# \text{ of Conf.}= \text{approx.} 1k - 2k \ (10k - 20k \text{ meas.)} \]
Pressures

\[ \frac{p}{T^4} \bigg|_{\beta_0}^{\beta} = \int_{\beta_0}^{\beta} d\beta' \Delta \]

It becomes larger than the normal case?

Possible interpretation: because of releasing degree of freedom of magnetic parts from the remnant of spatial confinement?

We need more calculations on larger volume, etc to achieve a final result.
Summary

◆ We have studied the sQGP physics via the lens of center vortex mechanism.

◆ Magnetic degrees of freedom are so important via the study of the gluon propagators after/before removal of vortices; conversely center vortices are still significant objects above $T_c$.

◆ Also we show a preliminary calculation of transport coefficients, in particular, shear viscosity, which is affected by the center vortices.

◆ Large contribution of magnetic vortices to EOS (preliminary); but we have to investigate volume dependence and so on.